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ON THE AMPLE SERVICE ASSUMPTION OF PALM'S THEOREM IN INVENTORY MODELING

by

Donald Gross

Serial T-433
3 October 1980

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The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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A key assumption of much of the continuous review inventory modeling work is that orders placed do not queue up, so that there is complete order crossing and hence order lead times are strictly independent random variables. This paper investigates the effects of this assumption (which is almost never true).

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1. Introduction

An appropriate inventory policy in many situations is a one-for-one ordering policy [continuous review (s,S) policy where $s = S-1$]. That is, when a demand for an item arises, an order is immediately placed for a replacement. It is desired to find, then, the optimal value of the safety stock needed to support such a policy so that there is a control on both stockout probability and inventory investment.

Such a policy is most often used for items which are expensive and important, so that inventory investment and shortages are significant factors. Also, most repairable item inventory models fall into the one-for-one ordering category, as failed items are usually dispatched immediately to a repair facility upon failure. The METRIC class of models [see Muckstadt (1973)], one of the most useful multi-echelon models currently available, uses such a policy.

A key factor in these types of models is often the "ample server assumption;" that is, orders to be filled or items to be repaired never queue up but go into "service" immediately. Statistically, this means that successive order replenishment times (or repair times if we are

talking about repairable items) are *independent*. This assumption allows one to take advantage of Palm's Theorem from queueing theory, which states that if demand is Poisson [or compound Poisson--see Feeney and Sherbrooke (1966)], and there are ample "servers," then regardless of the distribution of order replenishment times, the state probabilities depend on the replenishment time [see Hadley and Whiton (1963), pp. 209 ff., for example]. In fact, letting N represent the steady state number of orders outstanding, λ the mean demand rate assuming the Poisson distribution, and T the mean replenishment lead time,

$$\Pr(N=n) \equiv \pi(n) = \frac{(\lambda T)^n e^{-\lambda T}}{n!} . \tag{1}$$

If we denote the steady state on-hand inventory by Z and assume complete backordering, then we have $Z = S-N$ and

$$\Pr(Z=z) \equiv p(z) = \pi(S-z) . \tag{2}$$

Using this relationship, it is easy to set up cost equations in terms of the decision variable S to be minimized. Since a shortage cost in many cases may be hard to assess, a service level constraint is often used instead. Fill rate (the percentage of requests filled immediately from on-shelf inventory) is one such constraint in wide use. Denoting the fill rate by F, we have

$$\begin{aligned} F &= \frac{\lambda - \Pr(Z\leq 0)}{\lambda} \times 100 \\ &= [1 - \Pr(Z\leq 0)] \times 100 \\ &= \left[1 - \sum_{n=S}^{\infty} \pi(n)\right] \times 100 \\ &= \left[\sum_{n=0}^{S-1} \pi(n)\right] \times 100 . \end{aligned} \tag{3}$$

Now suppose there is not ample service in the order filling (or repair) process. The question we seek to answer is, "What effect does this have on the calculation of S and on the actual F to be realized?" After all, one might argue from the inventory manager's point of view

that if, on the average, an order takes τ time units to be received after being placed, what difference does it make if it spends part of its time waiting in a queue to be processed or if it goes into processing immediately? The answer lies in the fact that if queueing occurs, successive replenishment times are correlated and the distribution of $\pi(n)$ can be radically changed.

2. Ample Servers versus Single Server Cases

To see the effect of introducing correlation in successive order replenishment times, let us suppose that instead of a potentially infinite number of "order pickers" (or repair channels), there is only one. Further, let us assume that order filling times are exponentially distributed with mean rate μ. Equation (1) still suffices for the ample server case (with $\tau = 1/\mu$), and in terms of queueing notation we call this the $M/M/\infty$ model with mean arrival rate λ and mean service rate μ $(= 1/\tau)$.

For the single server case, we have an M/M/1 model, still with mean arrival rate λ, but with a mean service rate of $\mu \neq 1/\tau$. Here, τ is equal to the total expected waiting plus service time to process an order (which is usually denoted as W in standard queueing notation). Further, from M/M/1 queueing theory,

$$\tau = W = \frac{1}{\mu-\lambda} , \tag{4}$$

so that the μ to make the M/M/1 "equivalent" in terms of mean lead time to the $M/M/\infty$ is then [rewriting Equation (4)]

$$\mu = \frac{1+\lambda\tau}{\tau} . \tag{5}$$

Now the difference between the two systems can be clearly seen. Denoting the steady state probabilities for the ample server case by $\pi(n)$ [Equation (1)] and the single server case by $\pi_1(n)$, it can readily be shown [see, for example, Hillier and Lieberman (1980), p. 418] that

$$\pi_1(n) = \left(1 - \frac{\lambda\tau}{1+\lambda\tau}\right)\left(\frac{\lambda\tau}{1+\lambda\tau}\right)^n = \left(\frac{1}{1+\lambda\tau}\right)\left(\frac{\lambda\tau}{1+\lambda\tau}\right)^n . \qquad (6)$$

Note that in the ample server case, the steady state probabilities that n orders are outstanding are Poisson, while for the single server case the steady state probabilities are geometric, even though the mean number of orders outstanding is the same, namely, $\lambda\tau$, the mean leadtime demand. As we shall see later, in certain cases (certain values of $\lambda\tau$), sizable discrepancies in S and F can result from assuming an ample server situation when in reality there is only a single server, even though the mean replenishment times are the same.

3. Ample Servers versus Multiple Server Case

The $M/M/\infty$ and $M/M/1$ cases are the extremes. We consider now "equivalent" $M/M/c$ systems for comparison to $M/M/\infty$. The time in system (waiting plus service) for an $M/M/c$ queue is given as

$$\tau = W = \frac{1}{\mu} + \frac{\mu(\lambda/\mu)^c / (c-1)!(c\mu-\lambda)^2}{\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)} . \qquad (7)$$

It is now necessary to employ numerical solution techniques to find the desired μ which will enable the calculation of the $\pi_c(n)$. We know that

$$\frac{1}{\tau} < \mu < \frac{1+\lambda\tau}{\tau} ; \qquad (8)$$

that is, the resulting μ will be somewhere between the $M/M/\infty$ and $M/M/1$ cases. A Newton-Raphson procedure was easily employed to calculate μ , and once having done so, we have from queueing theory

$$\pi_c(n) = \begin{cases} \dfrac{\lambda^n}{n!\ \mu^n}\,\pi_c(0) , & n < c \\[2ex] \dfrac{\lambda^n}{c^{n-c}c!\ \mu^n}\,\pi_c(0) , & n \geq c \end{cases} , \qquad (9)$$

where

$$\pi_c(0) = \left[\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} .$$

Thus the resulting fill rate becomes

$$F = \left[\sum_{n=0}^{S-1} \pi_c(n)\right] \times 100 . \tag{10}$$

We can now compare the F's and S's obtained when using the ample server assumption in a situation where service is not truly ample; that is, for part of the replenishment leadtime items may wait in a queue.

4. Numerical Results

The calculations for S are performed by setting a desired fill rate level, say $\hat{F}$, and solving for the S such that

$$100 \sum_{n=0}^{S-1} \pi(n) \overset{\text{just}}{>} \hat{F} . \tag{11}$$

For the ample server case, $\pi_\infty(n)$ [from Equation (1)] is used in Equation (11), while for the "equivalent" c server case, $\pi_c(n)$ [from Equation (9)] is utilized. The respective S's obtained we denote by S_∞ and S_c.

If in reality, we truly had an M/M/c system, but were using the ample service assumption to calculate S, that is we stock S_∞, then the true fill rate $F(S_\infty, \pi_c)$ [call F_∞] is

$$F_\infty = \sum_{n=0}^{S_\infty - 1} \pi_c(n)$$

and may be less than $\hat{F}$, since $S_\infty < S_c$ for all reasonable values of c. It is this type of "error" which is of interest.

Another "error" of interest involves the expected average back-order level (which is sometimes used instead of fill rate as a service level constraint). For a safety stock level of S units, the expected average backorder level is

$$\bar{B} = \sum_{n=S}^{\infty} (n-S)\pi(n) = L - S - \sum_{n=0}^{S-1} (n-S)\pi(n) , \tag{12}$$

where L is the expected number of orders outstanding, that is,

$$L \equiv \sum_{n=0}^{\infty} n\pi(n) = \begin{cases} \lambda\tau , & \text{ample service ,} \\ \dfrac{\lambda}{\mu} + \left[\dfrac{(\lambda/\mu)^c \lambda\mu}{(c-1)!(c\mu-\lambda)^2}\right]\left[\displaystyle\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} , & c \text{ servers .} \end{cases} \tag{13}$$

Thus if we provision based on ample servers, that is, stock according to S_∞ , our expected average backorder level $\bar{B}(S_\infty,\pi_c)$ [call $\bar{B}_\infty$] is

$$\bar{B}_\infty = L_c - S_\infty - \sum_{n=0}^{S_\infty-1} (n-S_\infty)\pi_c(n) ,$$

whereas had we used the "correct" modeling assumptions, accounting for the fact that only c servers are available, our expected average back-order level would have been $\bar{B}(S_c,\pi_c)$ [call $\bar{B}_c$], namely,

$$\bar{B}_c = L_c - S_c - \sum_{n=0}^{S_c-1} (n-S_c)\pi_c(n) .$$

If in reality we had ample service, the expected average backorder level would have been $\bar{B}(S_\infty,\pi_\infty)$ [denote by $\bar{B}^*$], specifically,

$$\bar{B}^* = L_\infty - S_\infty - \sum_{n=0}^{S_\infty-1} (n-S_\infty)\pi_\infty(n) .$$

Table 1 shows the input and output quantities used for the numerical analyses. We calculate several possible "error" measures on fill rate and backorder level as defined in the table. For fill rate, we look at three quantities. First we compute the percent difference between the actual fill rate attained (F_∞) assuming ample service and using S_∞ and the fill rate we should have gotten (F_c) by stocking S_c had we correctly accounted for the fact that only c servers were available. This we call D_{F_∞}. Next we compute $D_{\hat{F}}$, the percent that the actual fill rate, F_∞, is below our goal $\hat{F}$. For the cases where $S_\infty = S_c$, there is no error and we set $D_{\hat{F}}$ to zero, since for these cases we always either achieve or exceed the goal $\hat{F}$. The final measure on fill rate we compute is D_{F*}, the percent difference between what we *think* we are achieving by assuming ample service $(F*)$ and what we are really achieving (F_∞).

For expected average backorder level, we compute two measures, namely, $D_{\overline{B}_\infty}$, the percent difference in $\overline{B}$ for a c server system if we stock under ample service conditions--that is, using S_∞ instead of the correct S_c --and $D_{\overline{B}*}$, the percent difference in the perceived (believing we have ample service) and actual (with c servers) $\overline{B}$'s.

Figures 1, 2, and 3 show the output of the cases considered, namely, for $\hat{F}$ of 80% (Figure 1), 90% (Figure 2), and 95% (Figure 3), we have computed the error measures for combinations of $\lambda\tau = .25$, .50, 1, 5, 10, 15, ..., 50, and $c = 1,3,5,10,15,20,25$. It appears that the larger errors occur for the larger values of $\lambda\tau$ and smaller values of c, as we would expect. Also, the three fill rate measures seem to track quite closely with each other. Note that the magnitude of the percent error for the backorder measures is much higher than that for the fill rate measures, with $D_{\overline{B}*}$ being an order of magnitude higher than $D_{\overline{B}_\infty}$, which itself is almost an order of magnitude higher than the D_F measures.

TABLE 1

FACTORS IN THE NUMERICAL ANALYSES

Symbol	Definition	Formula
	INPUT	
$\hat{F}$	Desired fill rate	Input
λt	Mean demand over a replenishment lead-time	Input
c	Number of servers (order "pickers" or repair channels)	Input
	OUTPUT	
S_∞	Safety stock required to achieve $\hat{F}$ if ample servers available	$\sum_{n=0}^{S_\infty-1} \pi_\infty(n) \text{ just } \geq \hat{F}$
S_c	Safety stock required to achieve $\hat{F}$ under c servers ($S_c \geq S_\infty$)	$\sum_{n=0}^{S_c-1} \pi_c(n) \text{ just } \geq \hat{F}$
F^*	Actual fill rate achieved using S_∞ if the true state of affairs is ample service	$F^* = \sum_{n=0}^{S_\infty-1} \pi_\infty(n)$
F_c	True fill rate for c servers stocking with S_c ($F_c \geq \hat{F}$)	$F_c = \sum_{n=0}^{S_c-1} \pi_c(n)$
F_∞	Actual fill rate achieved for c servers stocking under the assumption of ample service; i.e., using S_∞ ($F_\infty \leq F_c$)	$F_\infty = \sum_{n=0}^{S_\infty-1} \pi_c(n)$
D_{F_∞}	Percent difference actual fill rate is *below* correct fill rate when stocking for a c-server system but using the ample service assumptions	$D_{F_\infty} = \frac{F_c - F_\infty}{F_c} \times 100$
$D_{\hat{F}}$	Percent actual fill rate is *below* fill rate goal when stocking for a c-server system but using the ample service assumption	$D_{\hat{F}} = \max\left[0, \frac{\hat{F} - F_\infty}{\hat{F}} \times 100\right]$

TABLE 1--*continued*

Symbol	Definition	Formula
D_{F*}	Percent actual fill rate is *below* assumed fill rate when stocking for a c-server system but using the ample service assumptions	$D_{F*} = \frac{F^* - F_\infty}{F^*} \times 100$
$D_{\bar{B}_\infty}$	Percent *increase* in expected average backorder level when stocking for a c-server system but using ample service assumptions	$D_{\bar{B}_\infty} = \frac{\bar{B}_\infty - \bar{B}_c}{\bar{B}_c} \times 100$
$D_{\bar{B}*}$	Percent actual expected average backorder level is *above* assumed expected average backorder level when stocking for a c-server system but using ample service assumptions	$D_{\bar{B}*} = \frac{\bar{B}_\infty - \bar{B}_*}{\bar{B}*} \times 100$

All values for the error measures in Figures 1, 2, and 3 are given in percents so that, for example from Figure 2, for $\hat{F} = 90\%$, $\lambda\tau = 20$, $c = 5$, D_{F_∞} shows a 17.93% error, $D_{\hat{F}}$ a 17.47% error, and D_{F*} a 19.45% error, while $D_{\bar{B}_\infty}$ shows a 170.67% error and $D_{\bar{B}*}$ a 3,113.81% error! Of course, $D_{\bar{B}*}$ shows such large errors because if we think we have ample service, we expect a very low average backorder level (namely, 0.14 units) while in reality we have a level of 4.52 units, which is a large percentage change from 0.14. Had we correctly used the stocking criteria for five servers $(S_c = 45)$, then our expected average backorder level would have been 1.67 units. Perhaps in terms of backorder measures, one should also keep in mind the absolute error as the percentage error is distorted by the small "base" upon which it is calculated.

While errors are larger for larger values of $\lambda\tau$ and smaller values of c, there is not always strict monotonicity which, we believe, is due to the discrete process required in calculating S to satisfy the inequality constraint on fill rate goal $\hat{F}$. For example, from

FHAT= 85.

LAM*TAU	C	SC	SINF	FC	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBRINF	BBRSTR	DBRINF	DBRSTR
0.25	1	2	2	96.00	96.00	97.35	0.0	0.0	1.39	0.0100	0.0100	0.0023	0.0	334.42
0.25	3	2	2	97.35	97.35	97.35	0.0	0.0	0.00	0.0024	0.0024	0.0023	0.0	4.72
0.25	5	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.25	10	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	15	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	20	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	25	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.50	1	2	2	88.89	88.89	90.98	0.0	0.0	2.30	0.0556	0.0556	0.0163	0.0	240.28
0.50	3	2	2	91.00	91.00	90.98	0.0	0.0	-0.02	0.0179	0.0179	0.0163	0.0	9.47
0.50	5	2	2	90.98	90.98	90.93	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.09
0.50	10	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	15	2	2	90.98	90.90	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	20	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	25	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
1.00	1	3	3	87.50	87.50	91.97	0.0	0.0	4.86	0.1250	0.1250	0.0233	0.0	435.64
1.00	3	3	3	91.75	91.75	91.97	0.0	0.0	0.23	0.0389	0.0389	0.0233	0.0	66.55
1.00	5	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0239	0.0239	0.0233	0.0	2.28
1.00	10	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	15	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	20	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	25	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
3.00	1	7	6	86.65	62.20	91.61	5.13	3.29	10.27	0.4005	0.5339	0.0507	33.33	953.06
3.00	3	6	6	85.69	85.69	91.61	0.0	0.0	6.46	0.3009	0.3009	0.0507	0.0	493.48
3.00	5	6	6	89.83	89.83	91.61	0.0	0.0	1.94	0.1265	0.1265	0.0507	0.0	149.48
3.00	10	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0510	0.0510	0.0507	0.0	0.58
3.00	15	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
3.00	20	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
3.00	25	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
5.00	1	11	8	86.54	76.74	86.66	11.32	9.71	11.45	0.6729	1.1628	0.1221	72.80	852.29
5.00	3	10	8	86.36	78.72	86.66	8.86	7.39	9.17	0.5468	0.8535	0.1221	56.09	598.99
5.00	5	9	8	86.41	81.68	86.66	5.47	3.90	5.75	0.3913	0.5273	0.1221	34.74	331.79
5.00	10	8	8	86.65	86.65	86.66	0.0	0.0	0.01	0.1386	0.1386	0.1221	0.0	13.53
5.00	15	8	8	86.66	86.66	86.66	0.0	0.0	-0.00	0.1222	0.1222	0.1221	0.0	0.06
5.00	20	8	8	86.66	86.66	86.66	0.0	0.0	-0.00	0.1221	0.1221	0.1221	0.0	0.00
5.00	25	8	8	86.66	86.66	86.66	0.0	0.0	-0.00	0.1221	0.1221	0.1221	0.0	0.00
10.00	1	20	14	85.14	73.67	86.45	13.47	13.33	14.78	1.4864	2.6333	0.1869	77.16	1306.71
10.00	3	19	14	85.03	74.56	86.45	12.31	12.28	13.75	1.3393	2.2752	0.1869	69.88	1117.12
10.00	5	18	14	85.22	75.86	86.45	10.99	10.75	12.25	1.1322	1.8494	0.1869	63.36	889.38
10.00	10	16	14	87.52	81.06	86.45	7.38	4.64	6.23	0.5385	0.8171	0.1869	51.73	337.10
10.00	15	14	14	86.17	86.17	86.45	0.0	0.0	0.32	0.2628	0.2628	0.1869	0.0	40.57
10.00	20	14	14	86.45	86.45	86.45	0.0	0.0	-0.00	0.1890	0.1890	0.1869	0.0	1.11
10.00	25	14	14	86.45	86.45	86.45	0.0	0.0	-0.00	0.1870	0.1870	0.1869	0.0	0.02
15.00	1	30	20	85.57	72.49	87.52	15.29	14.71	17.17	2.1638	4.1259	0.2123	90.67	1843.46
15.00	3	29	20	85.56	73.04	87.52	14.63	14.07	16.54	2.0117	3.7546	0.2123	86.64	1668.59
15.00	5	28	20	85.79	73.83	87.52	13.95	13.15	15.65	1.7912	3.2991	0.2123	84.19	1454.03
15.00	10	25	20	86.29	76.76	87.52	11.05	9.70	12.30	1.2310	2.0872	0.2123	69.54	883.15
15.00	15	22	20	86.83	81.52	87.52	6.11	4.10	6.86	0.7142	1.0020	0.2123	40.29	371.97
15.00	20	20	20	86.69	86.69	87.52	0.0	0.0	0.95	0.3632	0.3632	0.2123	0.0	71.09
15.00	25	20	20	87.51	87.51	87.52	0.0	0.0	0.01	0.2218	0.2218	0.2123	0.0	4.46
20.00	1	39	26	85.08	71.88	88.78	15.52	15.44	19.04	2.9830	5.6250	0.2186	88.57	2472.85
20.00	3	38	26	85.05	72.27	88.78	15.03	14.98	18.60	2.8278	5.2465	0.2186	85.54	2299.76
20.00	5	37	26	85.20	72.81	88.78	14.55	14.34	17.99	2.6015	4.7806	0.2186	83.77	2086.65
20.00	10	34	26	85.52	74.73	88.78	12.62	12.08	15.83	2.0088	3.5056	0.2186	74.51	1503.45
20.00	15	31	26	86.15	77.72	88.78	9.79	8.56	12.46	1.3871	2.2320	0.2186	60.92	920.92
20.00	20	28	26	86.70	82.19	88.78	5.20	3.31	7.43	0.8456	1.1325	0.2186	33.93	417.99
20.00	25	26	26	87.24	87.24	88.78	0.0	0.0	1.74	0.4449	0.4449	0.2186	0.0	103.48

Figure 1.--Output for $\hat{F}$ = 85% .

FHAT= 85.

LAM*TAU	C	SC	SINF	FC	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBRINF	BBRSTR	DBRINF	DBRSTR
25.00	1	49	31	85.37	70.35	86.33	17.59	17.23	18.51	3.6584	7.4115	0.3152	102.59	2251.69
25.00	3	48	31	85.36	70.62	86.33	17.27	16.92	18.20	3.5016	7.0262	0.3152	100.66	2129.43
25.00	5	47	31	85.51	71.00	86.33	16.97	16.47	17.76	3.2685	6.5427	0.3152	100.17	1976.03
25.00	10	43	31	85.09	72.23	86.33	15.11	15.02	16.33	2.8050	5.2227	0.3152	86.19	1557.18
25.00	15	40	31	85.53	74.01	86.33	13.47	12.93	14.27	2.1518	3.8652	0.3152	79.63	1126.44
25.00	20	37	31	86.20	76.65	86.33	11.08	9.82	11.21	1.5065	2.5488	0.3152	69.19	708.76
25.00	25	33	31	85.00	80.54	86.33	5.26	5.25	6.71	1.0765	1.3973	0.3152	29.80	343.36
30.00	1	58	37	85.07	70.28	88.04	17.39	17.32	20.17	4.4790	8.9173	0.2953	99.09	2919.72
30.00	3	57	37	85.05	70.50	88.04	17.12	17.06	19.92	4.3209	8.5298	0.2953	97.41	2788.49
30.00	5	56	37	85.17	70.61	88.04	16.87	16.70	19.57	4.0843	8.0418	0.2953	96.89	2623.22
30.00	10	53	37	85.39	71.78	88.04	15.95	15.56	18.47	3.4743	6.7143	0.2953	93.25	2173.68
30.00	15	49	37	85.13	73.13	88.04	14.09	13.96	16.93	2.9440	5.3185	0.2953	80.66	1701.03
30.00	20	46	37	85.68	75.05	88.04	12.41	11.70	14.75	2.2500	3.9203	0.2953	74.23	1227.54
30.00	25	42	37	85.21	77.72	88.04	8.79	8.56	11.72	1.7316	2.6086	0.2953	50.65	783.35
35.00	1	68	42	85.27	69.37	86.31	18.65	18.39	19.63	5.1537	10.7206	0.3789	108.02	2729.34
35.00	3	67	42	85.27	69.54	86.31	18.45	18.19	19.44	4.9948	10.3297	0.3789	106.81	2626.18
35.00	5	66	42	85.38	69.77	86.31	18.28	17.91	19.16	4.7570	9.8362	0.3789	106.77	2495.93
35.00	10	62	42	85.11	70.50	86.31	17.16	17.06	18.32	4.2808	8.4814	0.3789	98.13	2138.40
35.00	15	59	42	85.44	71.45	86.31	16.37	15.94	17.22	3.6026	7.0648	0.3789	96.10	1764.51
35.00	20	55	42	85.27	72.74	86.31	14.69	14.42	15.72	3.0382	5.6221	0.3789	85.05	1383.78
35.00	25	52	42	85.85	74.50	86.31	13.21	12.35	13.68	2.3338	4.2047	0.3789	80.16	1009.68
40.00	1	77	48	85.06	69.43	88.04	18.37	18.31	21.14	5.9747	12.2268	0.3448	104.64	3446.18
40.00	3	76	48	85.06	69.59	88.04	18.19	18.12	20.95	5.8089	11.8252	0.3448	103.57	3329.70
40.00	5	75	48	85.15	69.79	88.04	18.03	17.89	20.73	5.5755	11.3395	0.3448	103.38	3188.84
40.00	10	72	48	85.31	70.40	88.04	17.48	17.17	20.03	4.9573	9.9896	0.3448	101.51	2797.33
40.00	15	68	48	85.17	71.24	88.04	16.36	16.19	19.09	4.4040	8.5413	0.3448	93.94	2377.26
40.00	20	65	48	85.55	72.27	88.04	15.53	14.98	17.92	3.6928	7.0901	0.3448	92.00	1956.36
40.00	25	61	48	85.45	73.64	88.04	13.83	13.37	16.36	3.1090	5.6338	0.3448	81.21	1534.00
45.00	1	87	53	85.22	68.80	86.72	19.27	19.05	20.66	6.6494	14.0385	0.4178	111.12	3259.73
45.00	3	86	53	85.23	68.94	86.72	19.12	18.90	20.51	6.4823	13.6337	0.4178	110.32	3162.
45.00	5	85	53	85.29	69.08	86.72	19.01	18.74	20.35	6.2572	13.1587	0.4178	110.30	3049.
45.00	10	81	53	85.09	69.58	86.72	18.24	18.15	19.77	5.7751	11.7876	0.4178	104.11	2721.[illegible]
45.00	15	78	53	85.36	70.20	86.72	17.75	17.41	19.04	5.0809	10.3361	0.4178	103.47	2374.14
45.00	20	74	53	85.27	71.00	86.72	16.73	16.46	18.12	4.4968	8.8496	0.4178	96.80	2017.90
45.00	25	70	53	85.17	72.02	86.72	15.44	15.27	16.95	3.8998	7.3563	0.4178	88.63	1660.52
50.00	1	96	58	85.06	68.29	85.51	19.71	19.66	20.14	7.4708	15.8551	0.4922	112.23	3121.15
50.00	3	95	58	85.06	68.40	85.51	19.59	19.53	20.01	7.3036	15.4492	0.4922	111.53	3038.69
50.00	5	94	58	85.11	68.51	85.51	19.51	19.40	19.88	7.0773	14.9719	0.4922	111.55	2941.72
50.00	10	91	58	85.26	68.92	85.51	19.17	18.92	19.41	6.4476	13.5947	0.4922	110.85	2661.92
50.00	15	87	58	85.16	69.44	85.51	18.45	18.30	18.79	5.8850	12.1174	0.4922	105.90	2361.79
50.00	20	83	58	85.05	70.05	85.51	17.63	17.59	18.08	5.3060	10.6288	0.4922	100.31	2059.36
50.00	25	80	58	85.41	70.82	85.51	17.08	16.68	17.18	4.5578	9.1158	0.4922	100.00	1751.98

Figure 1.--*continued.*

FHAT= 90.

LAM*TAU	L	SL	SINF	FL	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBRINF	BBRSTR	DBRINF	DBRSTR
0.25	1	2	2	96.00	96.00	97.35	0.0	0.0	1.39	0.0100	0.0100	0.0023	0.0	334.42
0.25	3	2	2	97.35	97.35	97.35	0.0	0.0	0.00	0.0024	0.0024	0.0023	0.0	4.72
0.25	5	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.25	10	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	15	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	20	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	25	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.50	1	3	2	96.30	88.89	90.98	7.69	1.23	2.30	0.0185	0.0556	0.0163	199.98	240.28
0.50	3	2	2	91.00	91.00	90.98	0.0	0.0	-0.02	0.0179	0.0179	0.0163	0.0	9.47
0.50	5	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.09
0.50	10	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	15	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	20	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
0.50	25	2	2	90.98	90.98	90.98	0.0	0.0	-0.00	0.0163	0.0163	0.0163	0.0	0.02
1.00	1	4	3	93.75	87.50	91.97	6.67	2.78	4.86	0.0625	0.1250	0.0233	100.00	435.64
1.00	3	3	3	91.75	91.75	91.97	0.0	0.0	0.23	0.0389	0.0389	0.0233	0.0	66.55
1.00	5	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0239	0.0239	0.0233	0.0	2.28
1.00	10	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	15	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	20	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
1.00	25	3	3	91.97	91.97	91.97	0.0	0.0	-0.00	0.0233	0.0233	0.0233	0.0	0.01
3.00	1	9	6	92.49	82.20	91.61	11.12	8.66	10.27	0.2253	0.5339	0.0507	137.04	953.06
3.00	3	7	6	90.30	85.69	91.61	5.11	4.79	6.46	0.2039	0.3009	0.0507	47.57	493.48
3.00	5	7	6	94.36	89.83	91.61	4.80	0.19	1.94	0.0701	0.1265	0.0507	80.38	149.48
3.00	10	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0510	0.0510	0.0507	0.0	0.58
3.00	15	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
3.00	20	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
3.00	25	6	6	91.61	91.61	91.61	0.0	0.0	-0.00	0.0507	0.0507	0.0507	0.0	0.03
5.00	1	13	9	90.65	80.62	93.19	11.07	10.42	13.49	0.4673	0.9690	0.0540	107.36	1694.00
5.00	3	12	9	91.26	82.96	93.19	9.09	7.82	10.97	0.3503	0.6832	0.0540	95.01	1164.78
5.00	5	11	9	92.51	86.41	93.19	6.60	3.99	7.28	0.2155	0.3913	0.0540	81.55	624.44
5.00	10	9	9	93.02	93.02	93.19	0.0	0.0	0.18	0.0688	0.0688	0.0540	0.0	27.45
5.00	15	9	9	93.19	93.19	93.19	0.0	0.0	0.00	0.0541	0.0541	0.0540	0.0	0.14
5.00	20	9	9	93.19	93.19	93.19	0.0	0.0	-0.00	0.0540	0.0540	0.0540	0.0	0.01
5.00	25	9	9	93.19	93.19	93.19	0.0	0.0	-0.00	0.0540	0.0540	0.0540	0.0	0.01
10.00	1	25	15	90.77	76.06	91.65	16.21	15.49	17.01	0.9230	2.3939	0.1035	159.37	2213.64
10.00	3	23	15	90.20	77.12	91.65	14.50	14.31	15.86	0.8765	2.0464	0.1035	133.46	1877.76
10.00	5	22	15	90.95	78.55	91.65	13.53	12.62	14.19	0.6931	1.6359	0.1035	136.04	1481.05
10.00	10	18	15	91.77	84.62	91.65	7.79	5.97	7.67	0.3549	0.6633	0.1035	86.91	541.08
10.00	15	15	15	90.94	90.94	91.65	0.0	0.0	0.78	0.1722	0.1722	0.1035	0.0	66.39
10.00	20	15	15	91.65	91.65	91.65	0.0	0.0	0.01	0.1055	0.1055	0.1035	0.0	1.94
10.00	25	15	15	91.65	91.65	91.65	0.0	0.0	-0.00	0.1035	0.1035	0.1035	0.0	0.04
15.00	1	36	21	90.21	74.21	91.70	17.73	17.54	19.07	1.4691	3.8680	0.1293	163.29	2890.72
15.00	3	35	21	90.47	74.85	91.70	17.27	16.83	18.38	1.3271	3.5031	0.1293	163.98	2608.61
15.00	5	33	21	90.30	75.75	91.70	16.11	15.83	17.40	1.2228	3.0566	0.1293	149.97	2263.36
15.00	10	28	21	90.01	79.09	91.70	12.14	12.13	13.76	0.8968	1.8780	0.1293	109.41	1352.08
15.00	15	24	21	90.61	84.40	91.70	6.86	6.23	7.97	0.5090	0.8459	0.1293	66.18	554.07
15.00	20	21	21	90.26	90.26	91.70	0.0	0.0	1.57	0.2658	0.2658	0.1293	0.0	105.53
15.00	25	21	21	91.66	91.66	91.70	0.0	0.0	0.04	0.1384	0.1384	0.1293	0.0	7.02
20.00	1	48	27	90.39	73.21	92.21	19.00	18.65	20.60	1.9229	5.3571	0.1407	178.60	3706.20
20.00	3	46	27	90.10	73.66	92.21	18.25	18.15	20.12	1.8728	4.9832	0.1407	166.08	3440.50
20.00	5	45	27	90.49	74.27	92.21	17.93	17.47	19.45	1.6712	4.5233	0.1407	170.67	3113.81
20.00	10	40	27	90.46	76.43	92.21	15.51	15.08	17.11	1.3230	3.2699	0.1407	147.15	2223.24
20.00	15	35	27	90.54	79.74	92.21	11.92	11.40	13.52	0.9480	2.0294	0.1407	114.08	1341.91
20.00	20	30	27	90.07	84.61	92.21	6.06	5.99	8.24	0.6313	0.9785	0.1407	55.00	595.25
20.00	25	27	27	90.08	90.08	92.21	0.0	0.0	2.31	0.3457	0.3457	0.1407	0.0	145.63

Figure 2.--Output for $\hat{F} = 90\%$.

FHAT= 90.

LAM*TAU	C	SC	SINF	FC	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBINF	BBRSTR	DBRINF	DBRSTR
25.00	1	59	33	90.11	72.55	92.85	19.45	19.34	21.82	2.4715	6.8523	0.1436	177.26	4671.29
25.00	3	58	33	90.28	72.93	92.85	19.22	18.97	21.46	2.3246	6.4734	0.1436	178.48	4407.48
25.00	5	56	33	90.19	73.41	92.85	18.61	18.44	20.94	2.2121	5.9991	0.1436	171.20	4077.17
25.00	10	51	33	90.15	74.97	92.85	16.84	16.70	19.26	1.8534	4.7087	0.1436	154.06	3178.70
25.00	15	46	33	90.21	77.18	92.85	14.44	14.24	16.88	1.4562	3.3935	0.1436	133.03	2262.88
25.00	20	41	33	90.28	80.41	92.85	10.94	10.66	13.41	1.0610	2.1390	0.1436	101.60	1389.40
25.00	25	37	33	91.10	85.00	92.85	6.69	5.55	8.45	0.6590	1.0765	0.1436	69.47	649.55
30.00	1	71	38	90.25	71.23	91.10	21.07	20.85	21.80	2.9246	8.6297	0.2063	195.08	4083.41
30.00	3	69	38	90.06	71.48	91.10	20.63	20.57	21.53	2.8732	8.2446	0.2063	186.95	3896.75
30.00	5	68	38	90.33	71.83	91.10	20.48	20.19	21.15	2.6625	7.7601	0.2063	191.45	3661.84
30.00	10	63	38	90.32	72.91	91.10	19.27	18.98	19.96	2.3017	6.4434	0.2063	179.95	3023.57
30.00	15	58	38	90.45	74.42	91.10	17.72	17.31	18.30	1.8893	5.0627	0.2063	167.97	2354.25
30.00	20	52	38	90.11	76.55	91.10	15.05	14.95	15.97	1.5540	3.6857	0.2063	137.18	1686.74
30.00	25	47	38	90.18	79.47	91.10	11.88	11.70	12.76	1.1494	2.4033	0.2063	109.09	1065.04
35.00	1	82	44	90.07	71.05	92.09	21.12	21.06	22.85	3.4740	10.1332	0.1948	191.69	5102.44
35.00	3	81	44	90.19	71.26	92.09	21.00	20.83	22.62	3.3250	9.7463	0.1948	193.12	4903.80
35.00	5	79	44	90.14	71.55	92.09	20.62	20.50	22.30	3.2095	9.2584	0.1948	188.47	4653.29
35.00	10	74	44	90.12	72.45	92.09	19.61	19.50	21.32	2.8402	7.9209	0.1948	178.88	3966.61
35.00	15	69	44	90.20	73.62	92.09	18.38	18.19	20.05	2.4241	6.5266	0.1948	169.23	3250.78
35.00	20	64	44	90.38	75.20	92.09	16.79	16.44	18.33	1.9842	5.1142	0.1948	157.75	2525.66
35.00	25	58	44	90.06	77.34	92.09	14.13	14.07	16.02	1.6393	3.7376	0.1948	128.00	1818.92
40.00	1	94	49	90.18	70.18	90.75	22.18	22.02	22.67	3.9266	11.9286	0.2523	203.79	4627.56
40.00	3	92	49	90.05	70.36	90.75	21.87	21.83	22.47	3.8697	11.5287	0.2523	197.92	4469.10
40.00	5	91	49	90.25	70.57	90.75	21.80	21.59	22.24	3.6608	11.0453	0.2523	201.72	4277.50
40.00	10	86	49	90.24	71.26	90.75	21.04	20.83	21.48	3.2941	9.7022	0.2523	194.53	3745.21
40.00	15	80	49	90.03	72.17	90.75	19.83	19.81	20.47	2.9596	8.2631	0.2523	179.19	3174.84
40.00	20	75	49	90.16	73.31	90.75	18.69	18.54	19.22	2.5160	6.8232	0.2523	171.19	2604.19
40.00	25	70	49	90.36	74.82	90.75	17.20	16.87	17.56	2.0601	5.3820	0.2523	161.24	2033.01
45.00	1	105	55	90.05	70.14	91.84	22.11	22.06	23.62	4.4768	13.4348	0.2314	200.10	5707.03
45.00	3	104	55	90.15	70.30	91.84	22.02	21.88	23.45	4.3212	13.0330	0.2314	201.60	5533.36
45.00	5	102	55	90.09	70.48	91.84	21.77	21.69	23.26	4.2157	12.5613	0.2314	197.97	5329.49
45.00	10	97	55	90.08	71.09	91.84	21.09	21.01	22.59	3.8414	11.2019	0.2314	191.61	4741.90
45.00	15	92	55	90.16	71.65	91.84	20.31	20.17	21.76	3.4133	9.7670	0.2314	186.15	4121.68
45.00	20	87	55	90.31	72.82	91.84	19.37	19.09	20.71	2.9572	8.2970	0.2314	180.57	3486.23
45.00	25	81	55	90.15	74.03	91.84	17.89	17.74	19.39	2.5865	6.8270	0.2314	163.95	2850.90
50.00	1	117	60	90.14	65.52	90.77	22.88	22.75	23.41	4.9291	15.2394	0.2835	209.17	5274.72
50.00	3	115	60	90.03	69.65	90.77	22.64	22.61	23.27	4.8714	14.8361	0.2835	204.56	5132.47
50.00	5	114	60	90.18	69.80	90.77	22.61	22.45	23.11	4.6674	14.3615	0.2835	207.70	4965.08
50.00	10	109	60	90.19	70.29	90.77	22.06	21.90	22.56	4.2921	12.9938	0.2835	202.73	4482.71
50.00	15	103	60	90.04	70.93	90.77	21.22	21.19	21.86	3.9508	11.5286	0.2835	191.81	3965.97
50.00	20	98	60	90.14	71.67	90.77	20.49	20.37	21.05	3.4973	10.0542	0.2835	187.48	3445.95
50.00	25	92	60	90.00	72.60	90.77	19.33	19.33	20.02	3.1229	8.5591	0.2835	174.08	2918.66

Figure 2.--*continued.*

FHAT= 95.

LAM*TAU	C	SC	SINF	FC	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBRINF	BBRSTR	DBRINF	DBRSTR
0.25	1	2	2	96.00	96.00	97.35	0.0	0.0	1.39	0.0100	0.0100	0.0023	0.0	334.42
0.25	3	2	2	97.35	97.35	97.35	0.0	0.0	0.00	0.0024	0.0024	0.0023	0.0	4.72
0.25	5	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.25	10	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	15	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	20	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.17
0.25	25	2	2	97.35	97.35	97.35	0.0	0.0	-0.00	0.0023	0.0023	0.0023	0.0	0.12
0.50	1	3	3	96.30	96.30	98.56	0.0	0.0	2.30	0.0185	0.0185	0.0019	0.0	855.24
0.50	3	3	3	98.51	98.51	98.56	0.0	0.0	0.05	0.0030	0.0030	0.0019	0.0	52.83
0.50	5	3	3	98.56	98.56	98.56	0.0	0.0	0.00	0.0020	0.0020	0.0019	0.0	0.69
0.50	10	3	3	98.56	98.56	98.56	0.0	0.0	-0.00	0.0019	0.0019	0.0019	0.0	0.25
0.50	15	3	3	98.56	98.56	98.56	0.0	0.0	-0.00	0.0019	0.0019	0.0019	0.0	0.25
0.50	20	3	3	98.56	98.56	98.56	0.0	0.0	-0.00	0.0019	0.0019	0.0019	0.0	0.25
0.50	25	3	3	98.56	98.56	98.56	0.0	0.0	-0.00	0.0019	0.0019	0.0019	0.0	0.25
1.00	1	5	4	96.88	93.75	98.10	3.23	1.32	4.44	0.0313	0.0625	0.0043	100.00	1337.19
1.00	3	4	4	97.36	97.36	98.10	0.0	0.0	0.76	0.0125	0.0125	0.0043	0.0	186.34
1.00	5	4	4	98.09	98.09	98.10	0.0	0.0	0.01	0.0048	0.0048	0.0043	0.0	9.69
1.00	10	4	4	98.10	98.10	98.10	0.0	0.0	-0.00	0.0044	0.0044	0.0043	0.0	0.07
1.00	15	4	4	98.10	98.10	98.10	0.0	0.0	-0.00	0.0044	0.0044	0.0043	0.0	0.07
1.00	20	4	4	98.10	98.10	98.10	0.0	0.0	-0.00	0.0044	0.0044	0.0043	0.0	0.07
1.00	25	4	4	98.10	98.10	98.10	0.0	0.0	-0.00	0.0044	0.0044	0.0043	0.0	0.07
3.00	1	11	7	95.78	86.65	96.65	9.53	8.79	10.34	0.1267	0.4005	0.0172	216.05	2228.92
3.00	3	9	7	95.55	90.30	96.65	5.49	4.95	6.57	0.0936	0.2039	0.0172	117.77	1085.89
3.00	5	8	7	96.88	94.36	96.65	2.59	0.67	2.37	0.0389	0.0701	0.0172	80.37	307.83
3.00	10	7	7	96.65	96.65	96.65	0.0	0.0	0.00	0.0175	0.0175	0.0172	0.0	1.55
3.00	15	7	7	96.65	96.65	96.65	0.0	0.0	-0.00	0.0172	0.0172	0.0172	0.0	0.11
3.00	20	7	7	96.65	96.65	96.65	0.0	0.0	-0.00	0.0172	0.0172	0.0172	0.0	0.11
3.00	25	7	7	96.65	96.65	96.65	0.0	0.0	-0.00	0.0172	0.0172	0.0172	0.0	0.13
5.00	1	17	10	95.49	83.85	96.82	12.19	11.74	13.39	0.2254	0.8075	0.0222	258.33	3539.76
5.00	3	15	10	95.52	86.36	96.82	9.59	9.09	10.80	0.1796	0.5468	0.0222	204.39	2364.67
5.00	5	13	10	95.88	89.91	96.82	6.22	5.36	7.13	0.1187	0.2904	0.0222	144.62	1209.00
5.00	10	10	10	96.53	96.53	96.82	0.0	0.0	0.29	0.0342	0.0342	0.0222	0.0	54.09
5.00	15	10	10	96.82	96.82	96.82	0.0	0.0	0.00	0.0223	0.0223	0.0222	0.0	0.32
5.00	20	10	10	96.82	96.82	96.82	0.0	0.0	-0.00	0.0222	0.0222	0.0222	0.0	0.03
5.00	25	10	10	96.82	96.82	96.82	0.0	0.0	-0.00	0.0222	0.0222	0.0222	0.0	0.03
10.00	1	32	16	95.26	78.24	95.13	17.87	17.65	17.75	0.4735	[illegible]	0.0547	359.50	[illegible]
10.00	3	30	16	95.33	79.42	95.13	16.69	16.40	16.51	0.4174	1.8406	0.0547	340.93	3263.15
10.00	5	27	16	95.10	81.11	95.13	14.71	14.62	14.73	[illegible]	1.4470	0.0547	285.59	2544.01
10.00	10	21	16	95.60	87.52	95.13	8.45	7.88	8.00	0.1899	0.5385	0.0547	183.60	883.94
10.00	15	17	16	96.11	94.06	95.13	2.13	0.99	1.12	0.0739	0.1128	0.0547	52.61	106.11
10.00	20	16	16	95.11	95.11	95.13	0.0	0.0	0.01	0.0566	0.0566	0.0547	0.0	3.44
10.00	25	16	16	95.13	95.13	95.13	0.0	0.0	-0.00	0.0548	0.0548	0.0547	0.0	0.07
15.00	1	47	23	95.18	77.34	96.73	18.75	18.59	20.05	0.7223	3.3996	0.0435	370.66	7720.16
15.00	3	45	23	95.24	78.11	96.73	17.99	17.78	19.25	0.6634	3.0495	0.0435	359.71	6914.89
15.00	5	42	23	95.12	79.18	96.73	16.75	16.65	18.14	0.6151	2.6238	0.0435	326.59	5935.50
15.00	10	35	23	95.23	83.07	96.73	12.77	12.56	14.12	0.4283	1.5205	0.0435	255.05	3397.65
15.00	15	28	23	95.23	88.88	96.73	6.67	6.44	8.11	0.2586	0.6030	0.0435	133.15	1287.01
15.00	20	24	23	96.18	94.78	96.73	1.45	0.23	2.01	0.1042	0.1424	0.0435	36.64	227.52
15.00	25	23	23	96.64	96.64	96.73	0.0	0.0	0.09	0.0511	0.0511	0.0435	0.0	17.48
20.00	1	64	29	95.14	75.70	96.57	20.43	20.31	21.60	0.9711	4.8591	0.0539	400.38	8910.86
20.00	3	60	29	95.19	76.24	96.57	19.91	19.75	21.05	0.9105	4.4954	0.0539	393.73	8236.45
20.00	5	57	29	95.11	76.97	96.57	19.07	18.98	20.30	0.8604	4.0496	0.0539	370.68	7409.76
20.00	10	50	29	95.24	79.49	96.57	16.54	16.32	17.68	0.6596	2.8450	0.0539	331.31	5175.83
20.00	15	42	29	95.14	83.25	96.57	12.49	12.37	13.79	0.4871	1.6778	0.0539	244.46	3011.35
20.00	20	35	29	95.22	88.51	96.57	7.05	6.83	8.34	0.3041	0.7306	0.0539	140.23	1254.92
20.00	25	30	29	95.35	94.01	96.57	1.40	1.04	2.65	0.1623	0.2088	0.0539	28.67	287.18

Figure 3.--Output for $\hat{F}$ = 95% .

FHAT= 95.

LAM*TAU	C	SC	SINF	FC	FINF	FSTAR	DFINF	DFHAT	DFSTAR	BBARC	BBRINF	BBRSTR	DBRINF	DBRSTR
25.00	1	77	34	95.12	73.64	95.02	22.58	22.48	22.50	1.2200	6.5888	0.0938	440.06	6921.17
25.00	3	75	34	95.15	74.02	95.02	22.21	22.09	22.10	1.1584	6.2136	0.0938	436.38	6521.41
25.00	5	72	34	95.10	74.54	95.02	21.62	21.54	21.56	1.1050	5.7444	0.0938	419.84	6021.39
25.00	10	65	34	95.23	76.23	95.02	19.95	19.76	19.78	0.8975	4.4710	0.0938	398.19	4664.45
25.00	15	57	34	95.21	78.62	95.02	17.43	17.24	17.26	0.7118	3.1796	0.0938	346.71	3288.31
25.00	20	49	34	95.18	82.05	95.02	13.79	13.63	13.65	0.5263	1.9595	0.0938	272.30	1908.13
25.00	25	42	34	95.36	86.84	95.02	8.94	8.59	8.61	0.3329	0.9449	0.0938	183.79	906.88
30.00	1	92	40	95.10	73.06	95.37	23.18	23.09	23.40	1.4690	8.0819	0.0952	450.17	8392.16
30.00	3	90	40	95.11	73.36	95.37	22.89	22.78	23.08	1.4068	7.7026	0.0952	447.54	7993.55
30.00	5	87	40	95.09	73.77	95.37	22.42	22.35	22.65	1.3523	7.2259	0.0952	434.35	7492.72
30.00	10	80	40	95.19	75.06	95.37	21.16	20.99	21.30	1.1429	5.9340	0.0952	419.20	6135.24
30.00	15	72	40	95.21	76.82	95.37	19.31	19.13	19.45	0.9476	4.5875	0.0952	384.09	4720.36
30.00	20	64	40	95.28	79.27	95.37	16.81	16.56	16.89	0.7413	3.2579	0.0952	339.50	3323.28
30.00	25	56	40	95.30	82.58	95.37	13.36	13.08	13.42	0.5498	2.0399	0.0952	271.03	2043.50
35.00	1	107	46	95.09	72.63	95.75	23.62	23.54	24.14	1.7178	9.5781	0.0938	457.59	10111.64
35.00	3	105	46	95.12	72.88	95.75	23.38	23.28	23.89	1.6550	9.1959	0.0938	455.63	9704.20
35.00	5	102	46	95.08	73.22	95.75	22.99	22.93	23.53	1.5997	8.7145	0.0938	444.76	9190.95
35.00	10	94	46	95.01	74.27	95.75	21.83	21.82	22.43	1.4335	7.3974	0.0938	416.02	7786.69
35.00	15	86	46	95.00	75.63	95.75	20.39	20.39	21.01	1.2361	6.0294	0.0938	387.78	6328.25
35.00	20	78	46	95.04	77.44	95.75	18.52	18.48	19.12	1.0227	4.6522	0.0938	354.89	4859.91
35.00	25	70	46	95.10	79.85	95.75	16.03	15.94	16.60	0.8089	3.3225	0.0938	310.75	3442.26
40.00	1	122	52	95.08	72.31	96.13	23.95	23.89	24.78	1.9668	11.0768	0.0906	463.20	12123.13
40.00	3	120	52	95.11	72.53	96.13	23.74	23.65	24.55	1.9008	10.6833	0.0906	462.04	11688.82
40.00	5	117	52	95.08	72.81	96.13	23.42	23.36	24.26	1.8477	10.2075	0.0906	452.44	11163.81
40.00	10	109	52	95.01	73.67	96.13	22.47	22.46	23.36	1.6831	8.8886	0.0906	428.12	9708.43
40.00	15	101	52	95.03	74.81	96.13	21.28	21.26	22.18	1.4761	7.4815	0.0906	406.84	8155.77
40.00	20	93	52	95.07	76.21	96.13	19.83	19.78	20.72	1.2611	6.0813	0.0906	382.23	6610.59
40.00	25	85	52	95.15	78.04	96.13	17.97	17.85	18.81	1.0376	4.6921	0.0906	352.21	5077.64
45.00	1	137	57	95.08	71.43	95.27	24.87	24.81	25.03	2.2157	12.8570	0.1215	480.27	10478.73
45.00	3	135	57	95.10	71.61	95.27	24.70	24.62	24.83	2.1490	12.4588	0.1215	479.75	10151.07
45.00	5	132	57	95.06	71.82	95.27	24.45	24.40	24.62	2.0997	11.9911	0.1215	471.10	9766.30
45.00	10	124	57	95.02	72.52	95.27	23.67	23.66	23.88	1.9304	10.6453	0.1215	451.46	8658.98
45.00	15	116	57	95.02	73.40	95.27	22.75	22.73	22.95	1.7257	9.2275	0.1215	434.72	7492.35
45.00	20	108	57	95.08	74.51	95.27	21.63	21.57	21.79	1.5026	7.7789	0.1215	417.70	6300.47
45.00	25	100	57	95.16	75.90	95.27	20.24	20.11	20.33	1.2726	6.3359	0.1215	397.86	5113.17
50.00	1	152	63	95.07	71.28	95.76	25.02	24.97	25.56	2.4646	14.3605	0.1133	482.68	12575.11
50.00	3	150	63	95.09	71.44	95.76	24.87	24.80	25.39	2.3977	13.9617	0.1133	482.29	12223.15
50.00	5	147	63	95.06	71.62	95.76	24.65	24.61	25.20	2.3480	13.4922	0.1133	474.63	11808.74
50.00	10	139	63	95.02	72.24	95.76	23.97	23.96	24.56	2.1782	12.1418	0.1133	457.41	10616.83
50.00	15	131	63	95.04	73.02	95.76	23.17	23.14	23.75	1.9669	10.6987	0.1133	443.95	9343.07
50.00	20	123	63	95.08	73.93	95.76	22.24	22.17	22.79	1.7457	9.2500	0.1133	429.88	8064.39
50.00	25	114	63	95.00	75.07	95.76	20.98	20.98	21.60	1.5614	7.7871	0.1133	398.72	6773.20

Figure 3.--*continued.*

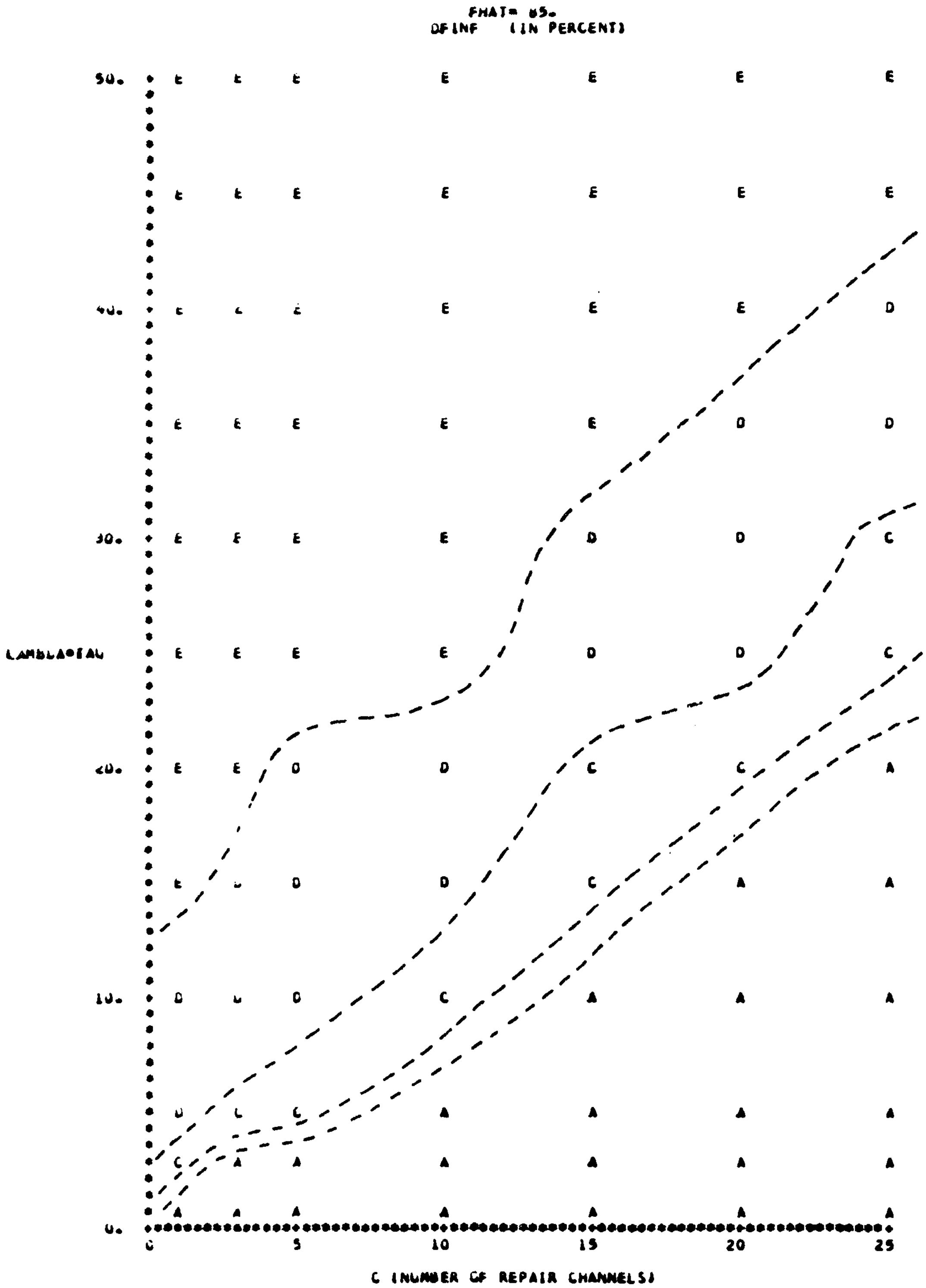
Figure 2, $\lambda\tau = 25$, we see that the percentage error in $D_{\bar{B}_\infty}$ increases slightly when going from $C = 1$ to $C = 3$, although this is a rather rare situation and the general trend is decreasing. The "nonmonotonicity" is somewhat more common when fixing c and observing the errors as $\lambda\tau$ increases. Again, even in cases where there is not strict monotonicity, the violations are small and there still remains a general trend.

Figures 4 through 18 show graphs of the error ranges of the D_F and $D_{\bar{B}}$ measures on the $\lambda\tau$ versus c space. A definite pattern emerges even though in a few cases the nonmonotonicity shows up. The error band lines are purposely plotted as "fuzzy," since the grid is not fine enough to obtain precise boundaries.

These graphs do show clearly that for large $\lambda\tau$ and small c , the percentage error can be sizable. Also, errors for comparable cases (same $\lambda\tau$ and c) become larger as $\hat{F}$ is increased. This can be seen by looking at comparable measures for the three $\hat{F}$ situations; for example, by comparing Figures 4, 9, and 14, or Figures 7, 12, and 17, and so forth.

While the general direction of large errors (larger $\lambda\tau$, smaller c , larger $\hat{F}$) may not be surprising, the actual magnitude might be. Certainly, one should give careful thought prior to employing the ample service assumption.

Figures 19 through 24 give plots of D_{F_∞} and $D_{\bar{B}_\infty}$ versus $\lambda\tau$ for various c values, and $\hat{F}$'s of 85%, 90%, and 95%, respectively. These sets of curves can be used to find the error (or approximate error if interpolation is necessary) in assuming ample service when in reality it is not. Keep in mind that these results assume exponential lead times in the nonample service model, and that the stockage criteria are based on fill rate control.



KEY: DFINF
A: - 0. %
B: 0.- 5. %
C: 5.- 10. %
D: 10.- 15. %
E: 15.- 20. %
F: 20.- %

Figure 4.--D_{F_∞} for $\hat{F}$ = 85% .

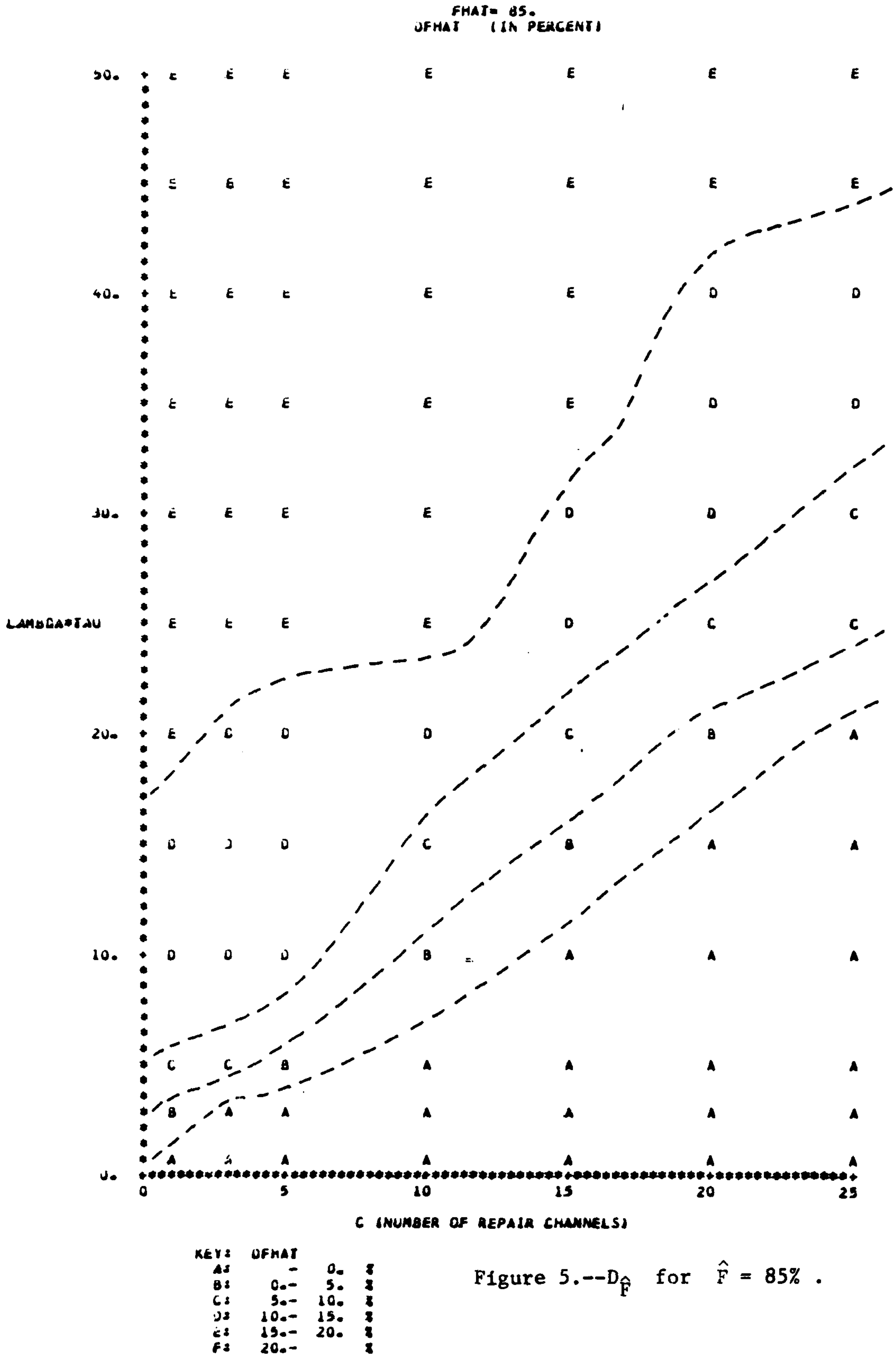


Figure 5.--$D_{\hat{F}}$ for $\hat{F}$ = 85% .

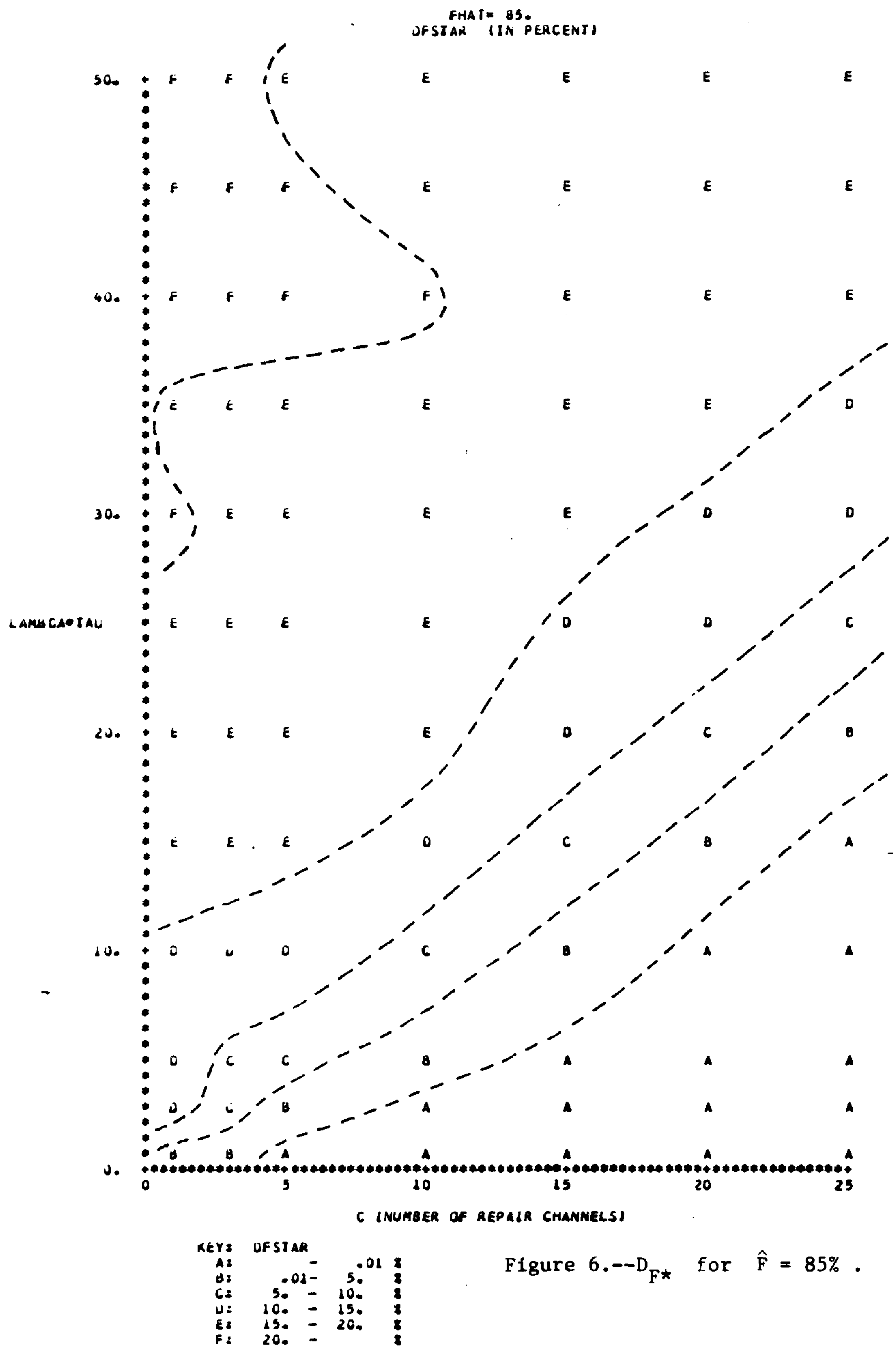
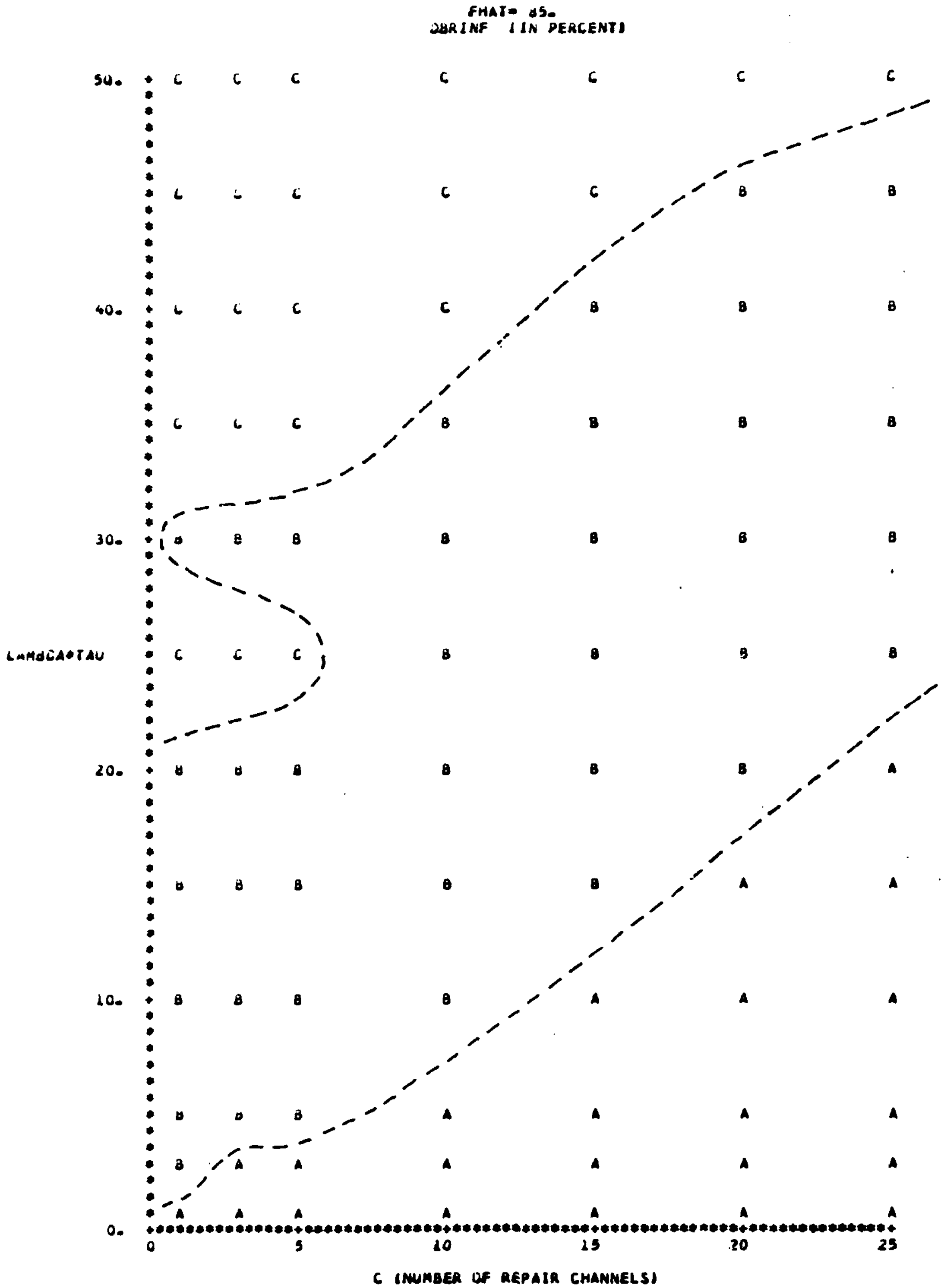


Figure 6.--D_{F*} for $\hat{F}$ = 85% .



KEY: DBRINF
A: - 0. %
B: 0.- 100. %
C: 100.- 200. %
D: 200.- 300. %
E: 300.- 400. %
F. 400.- %

Figure 7.--$D_{\overline{B}_{\infty}}$ for $\hat{F}$ = 85% .

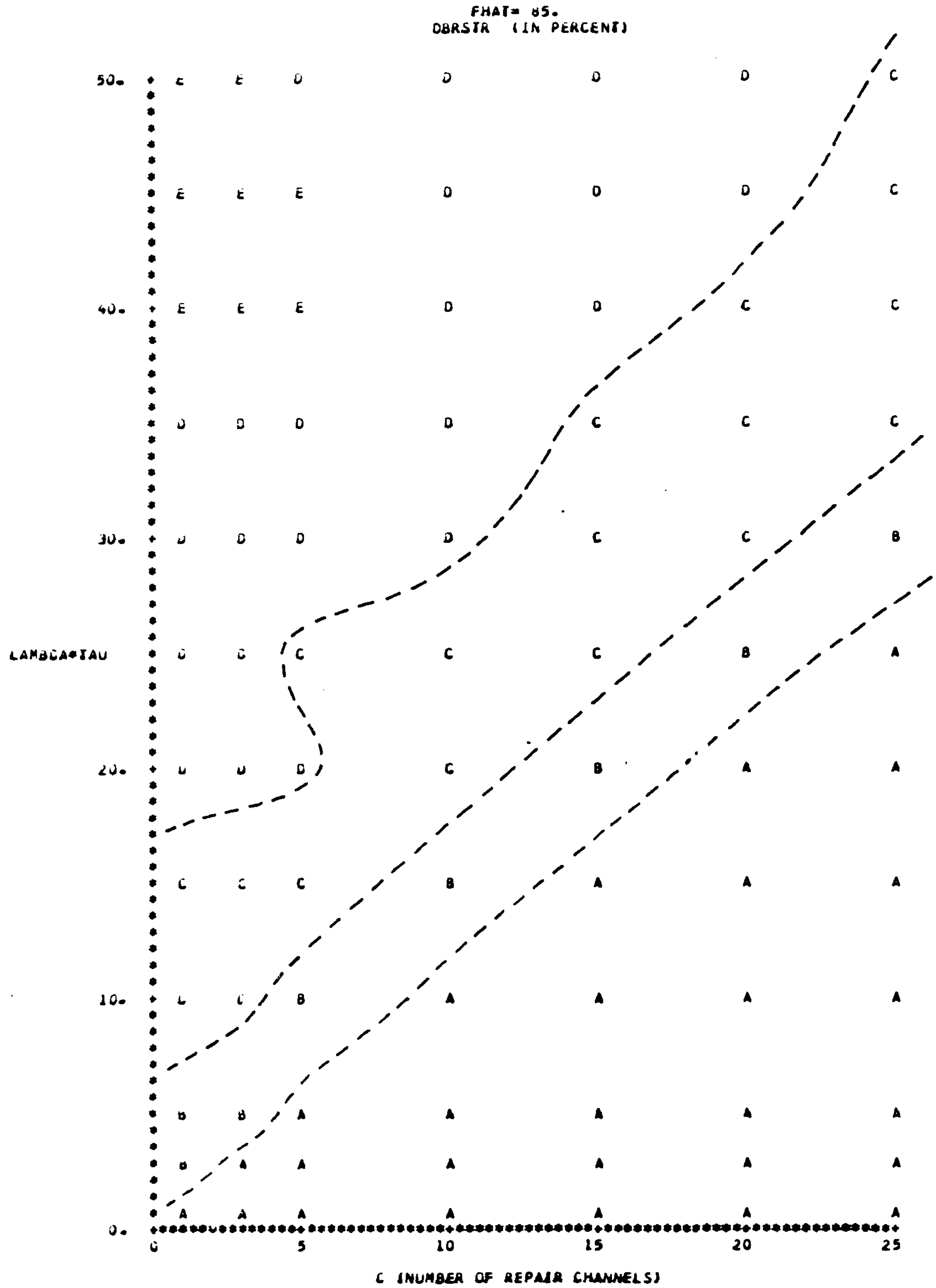
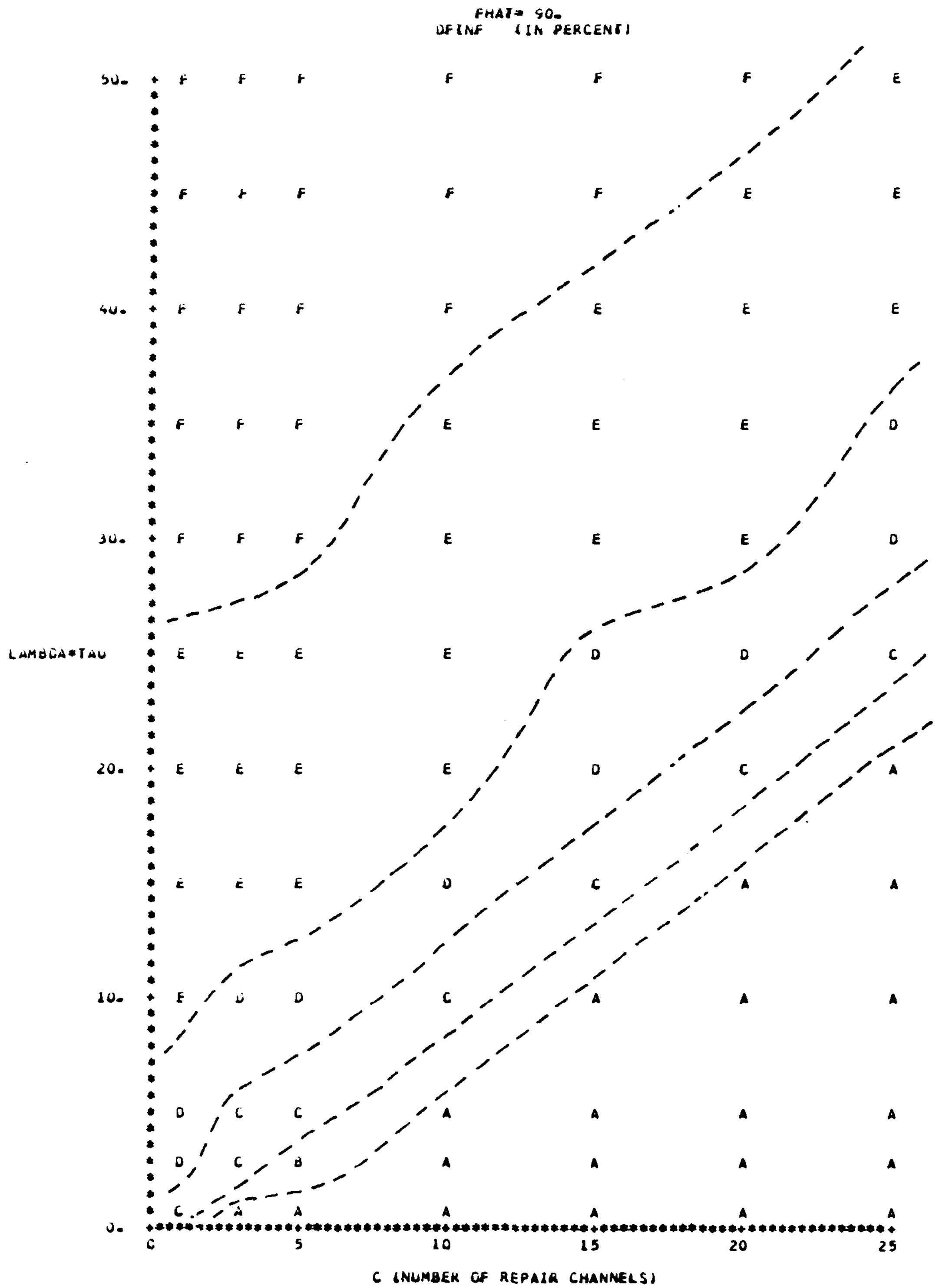


Figure 8.--$D_{\overline{B}*}$ for $\hat{F}$ = 85% .



KEY. DFINF
A: - 0. %
B: 0.- 5. %
C: 5.- 10. %
D: 10.- 15. %
E: 15.- 20. %
F: 20.- %

Figure 9.--D_{F_∞} for $\hat{F}$ = 90% .

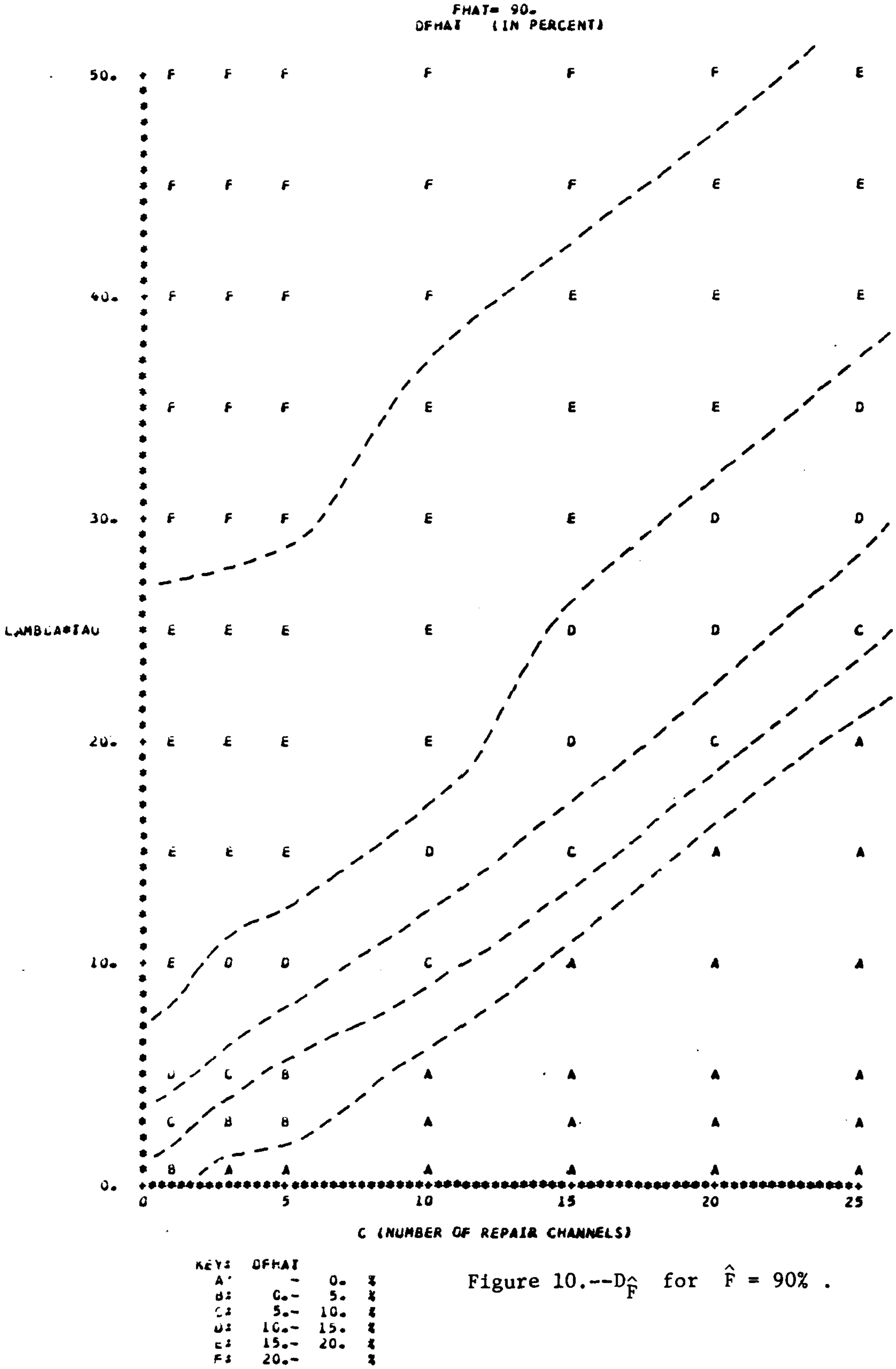
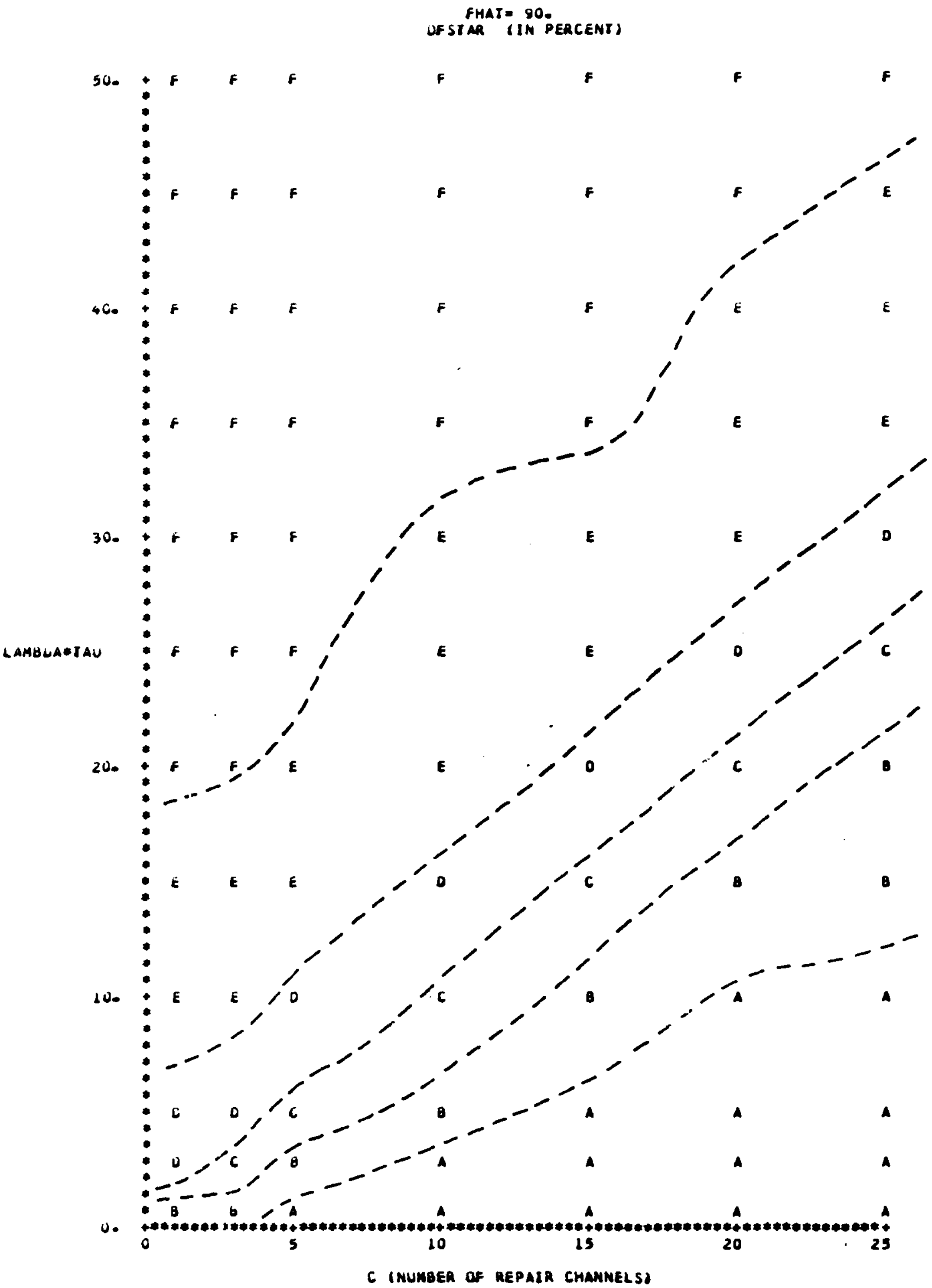


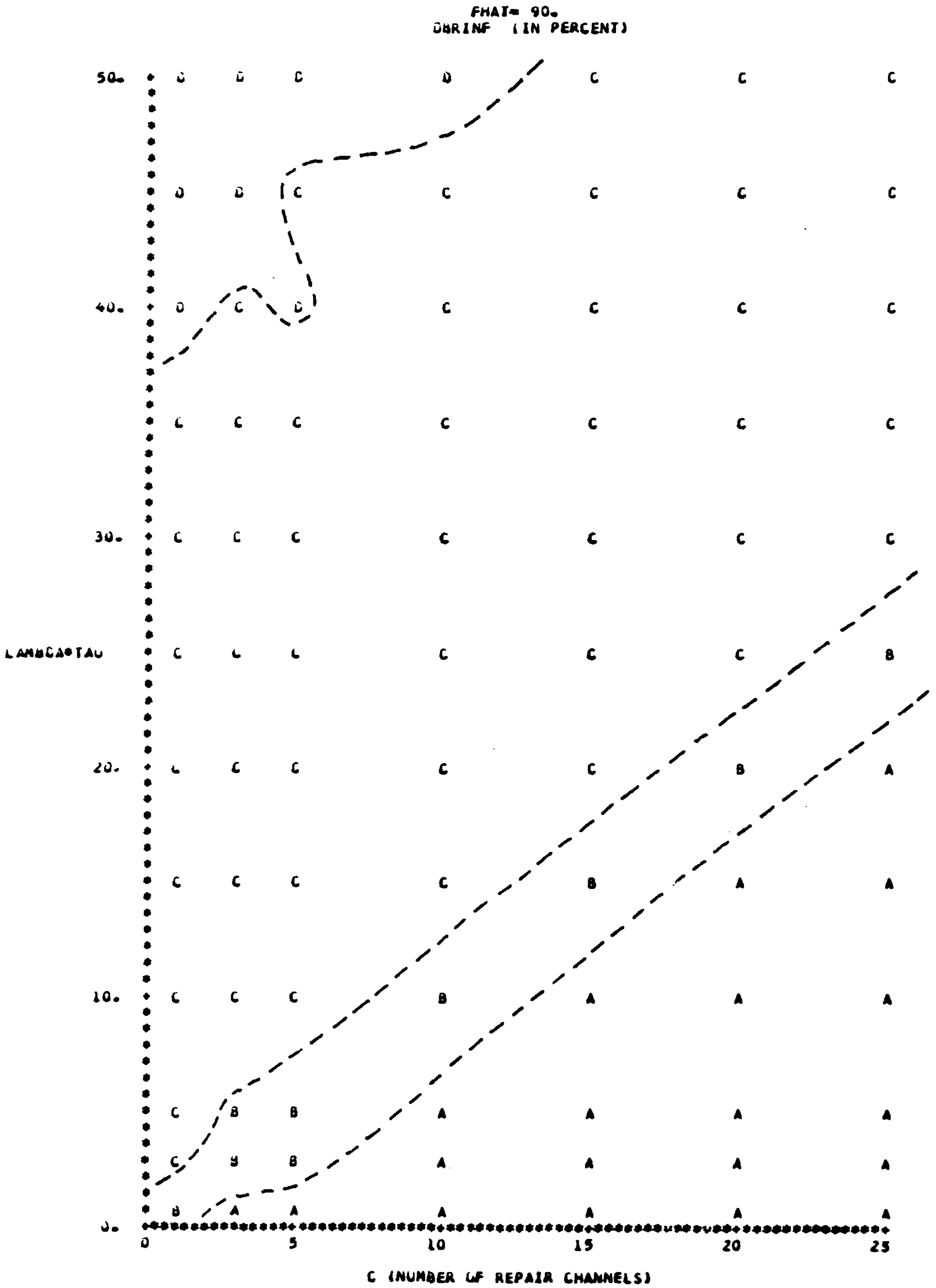
Figure 10.--$D_{\hat{F}}$ for $\hat{F}$ = 90% .



KEY: DFSTAR

A:		-	.01	%
B:	.01	-	5.	%
C:	5.	-	10.	%
D:	10.	-	15.	%
E:	15.	-	20.	%
F:	20.	-		%

Figure 11.--D_{F*} for $\hat{F}$ = 90% .



KEY: DBRINF
A: - 0. %
B: 0.- 100. %
C: 100.- 200. %
D: 200.- 300. %
E: 300.- 400. %
F: 400.- %

Figure 12.--$D_{\overline{B}_\infty}$ for $\hat{F}$ = 90% .

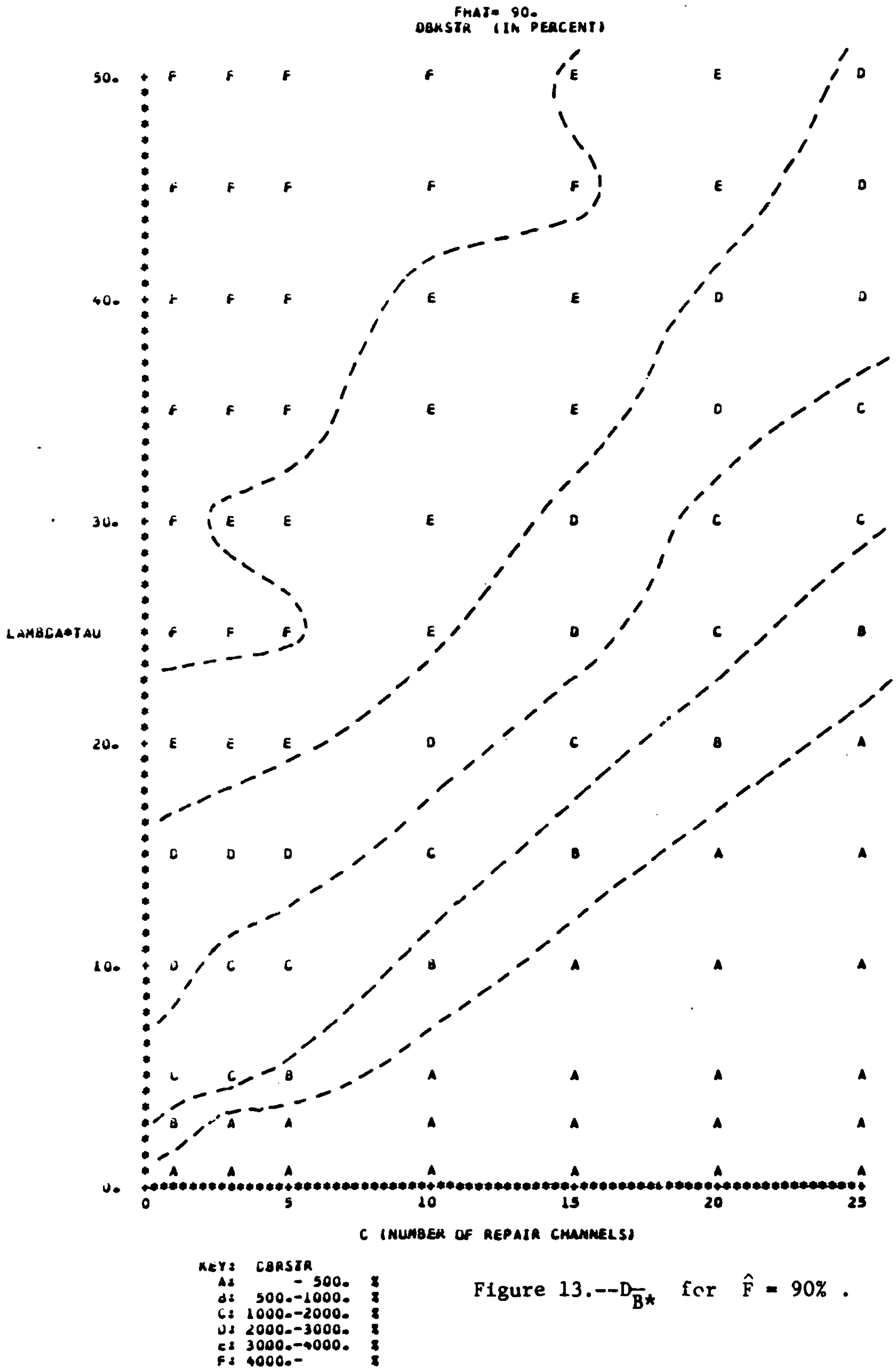
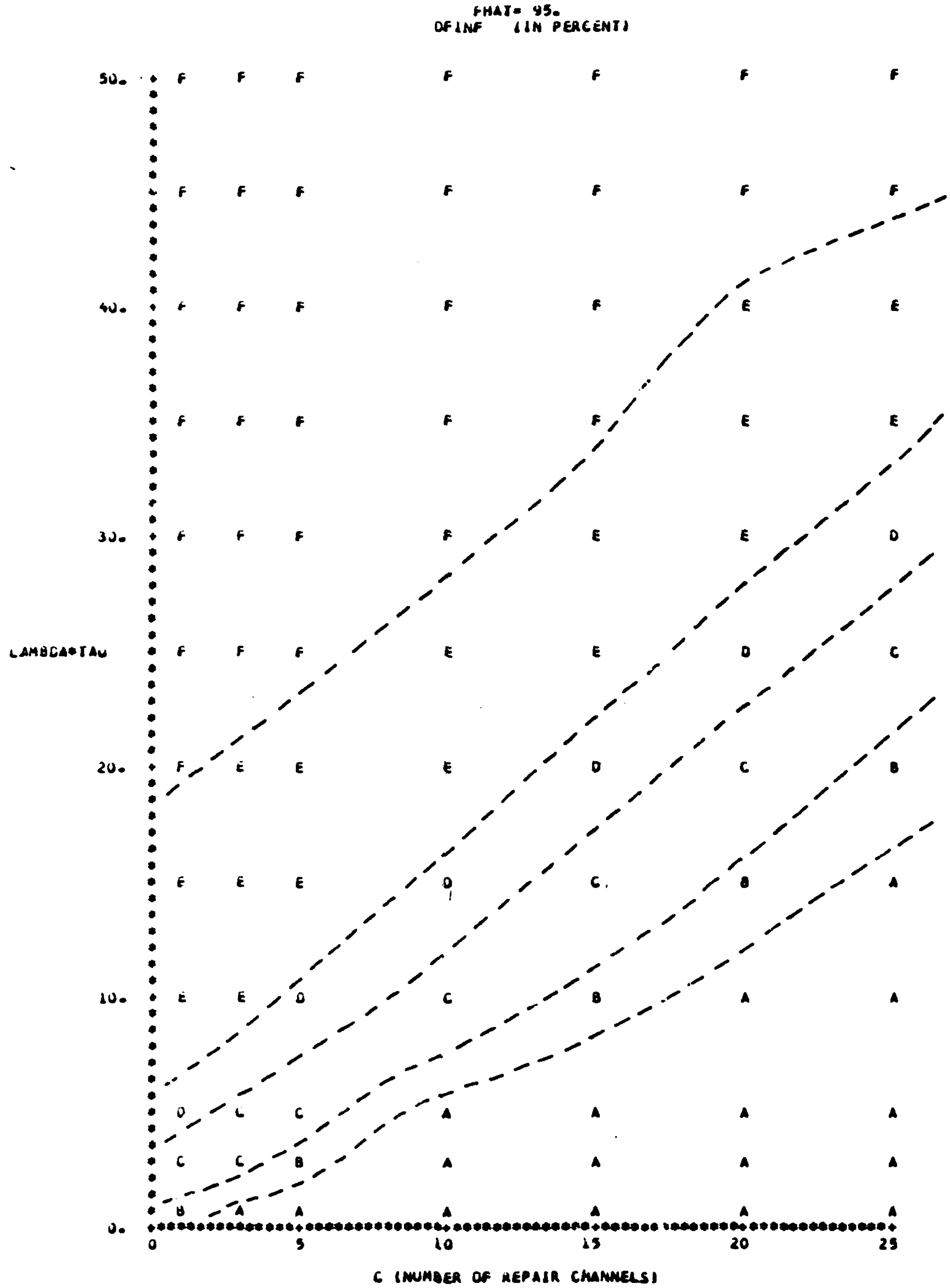
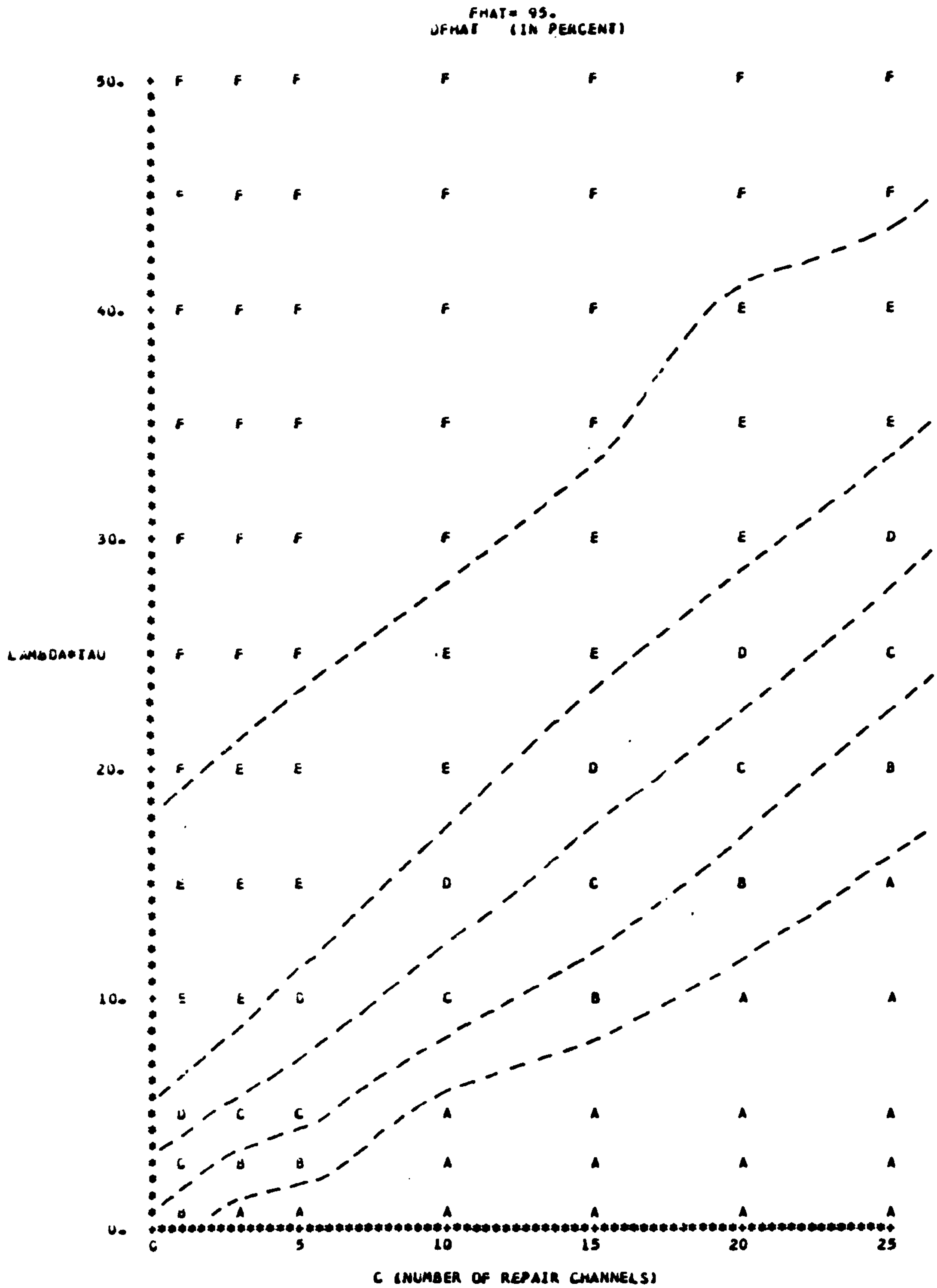


Figure 13.--$D_{\overline{B}*}$ for $\hat{F} = 90\%$.



KEY: DFINF
A: - 0. %
B: 0.- 5. %
C: .5.- 10. %
D: 10.- 15. %
E: 15.- 20. %
F: 20.- %

Figure 14.--D_{F_∞} for $\hat{F} = 95\%$.



KEY: DFHAT

A:	-	0.	%
B:	0.-	5.	%
C:	5.-	10.	%
D:	10.-	15.	%
E:	15.-	20.	%
F:	20.-		%

Figure 15.--$D_{\hat{F}}$ for $\hat{F}$ = 95% .

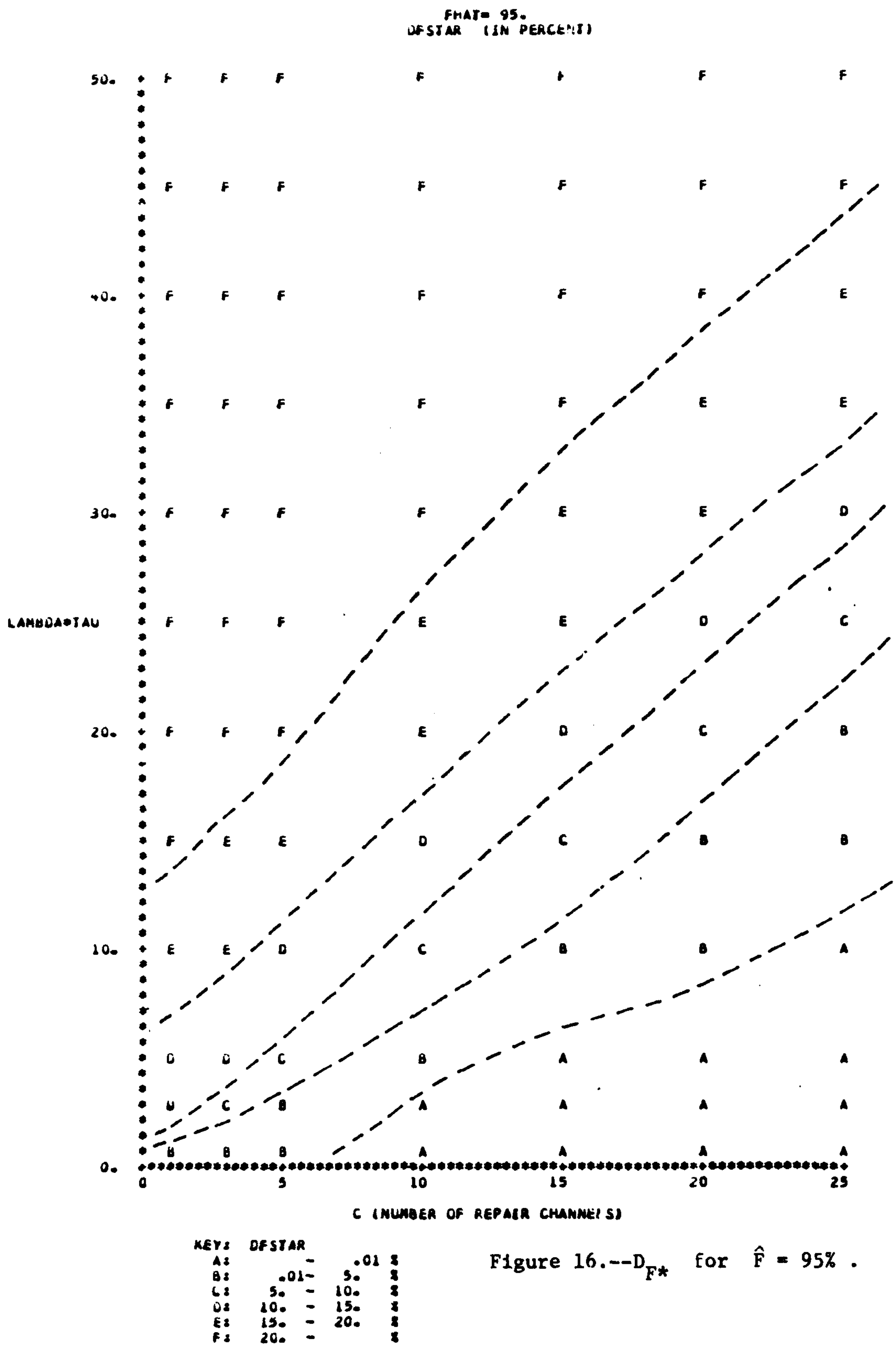
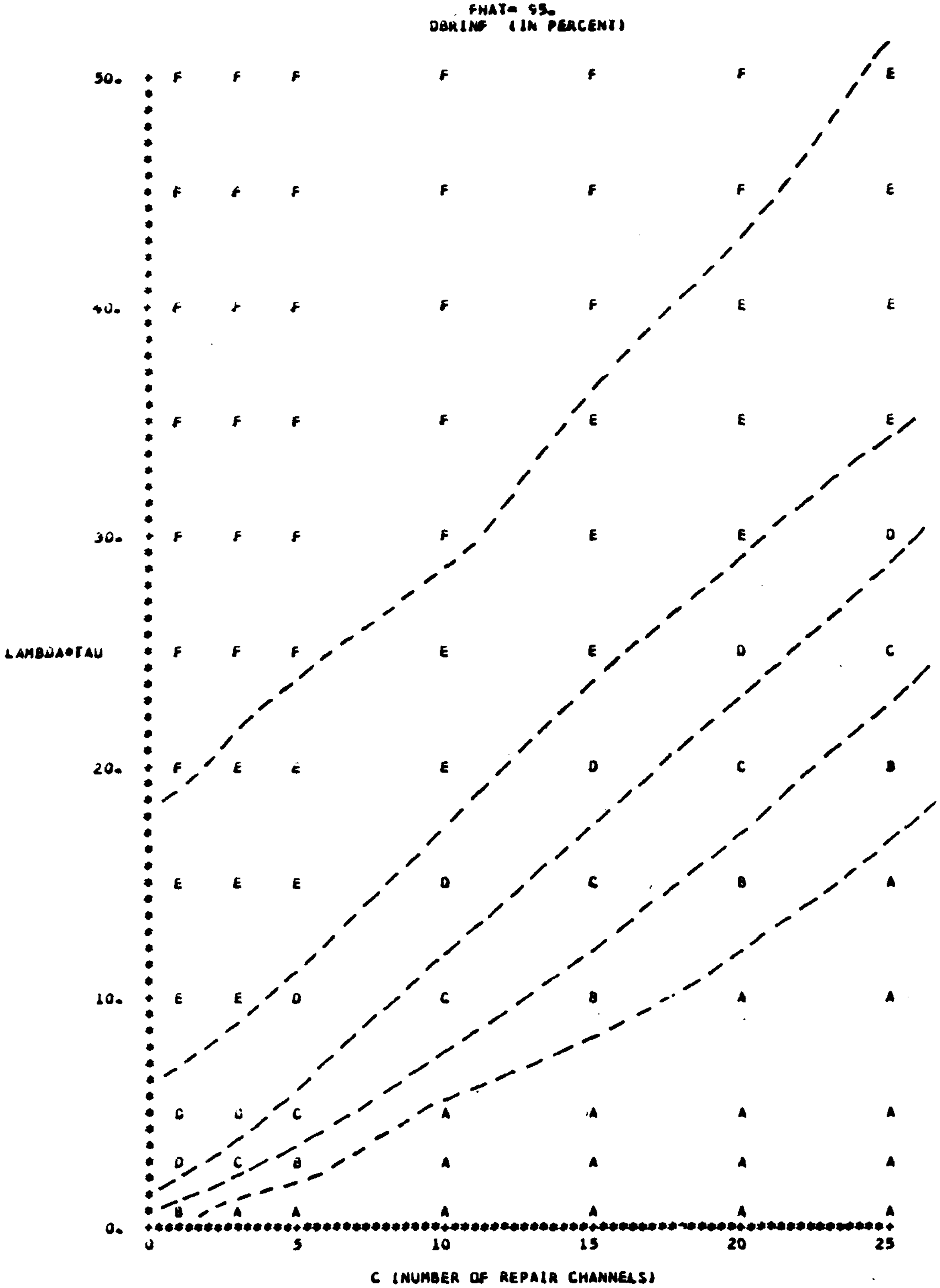
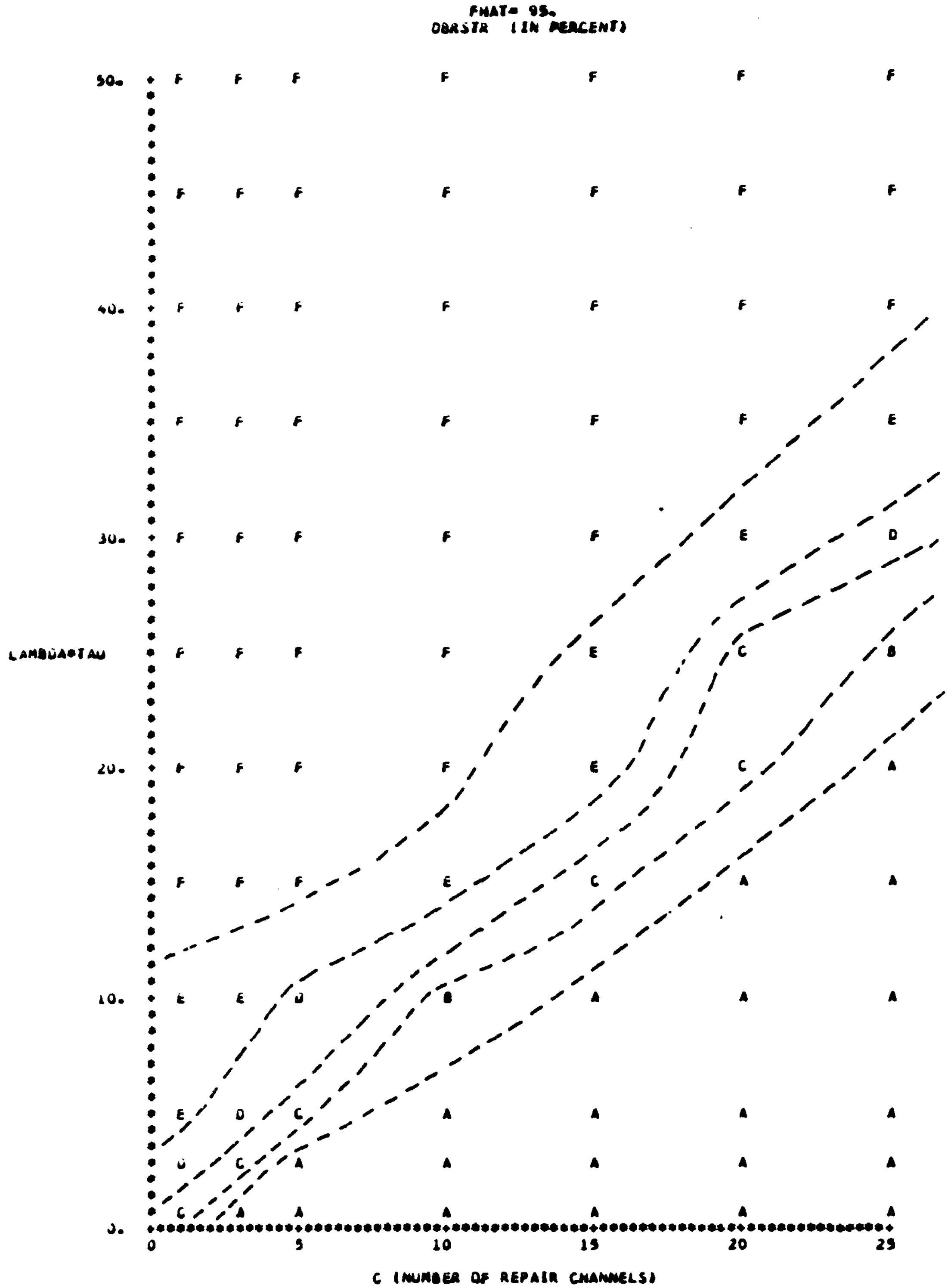


Figure 16.--D_{F*} for $\hat{F}$ = 95% .



KEY: DBRINF
A: - 0. %
B: 0.- 100. %
C: 100.- 200. %
D: 200.- 300. %
E: 300.- 400. %
F: 400.- %

Figure 17.--$D_{\overline{B}_\infty}$ for $\hat{F}$ = 95% .



KEYS DBRSTR
A: - 500. $
B: 500.-1000. $
C: 1000.-2000. $
D: 2000.-3000. $
E: 3000.-4000. $
F: 4000.- $

Figure 18.--$D_{\bar{B}*}$ for $\hat{F}$ = 95% .

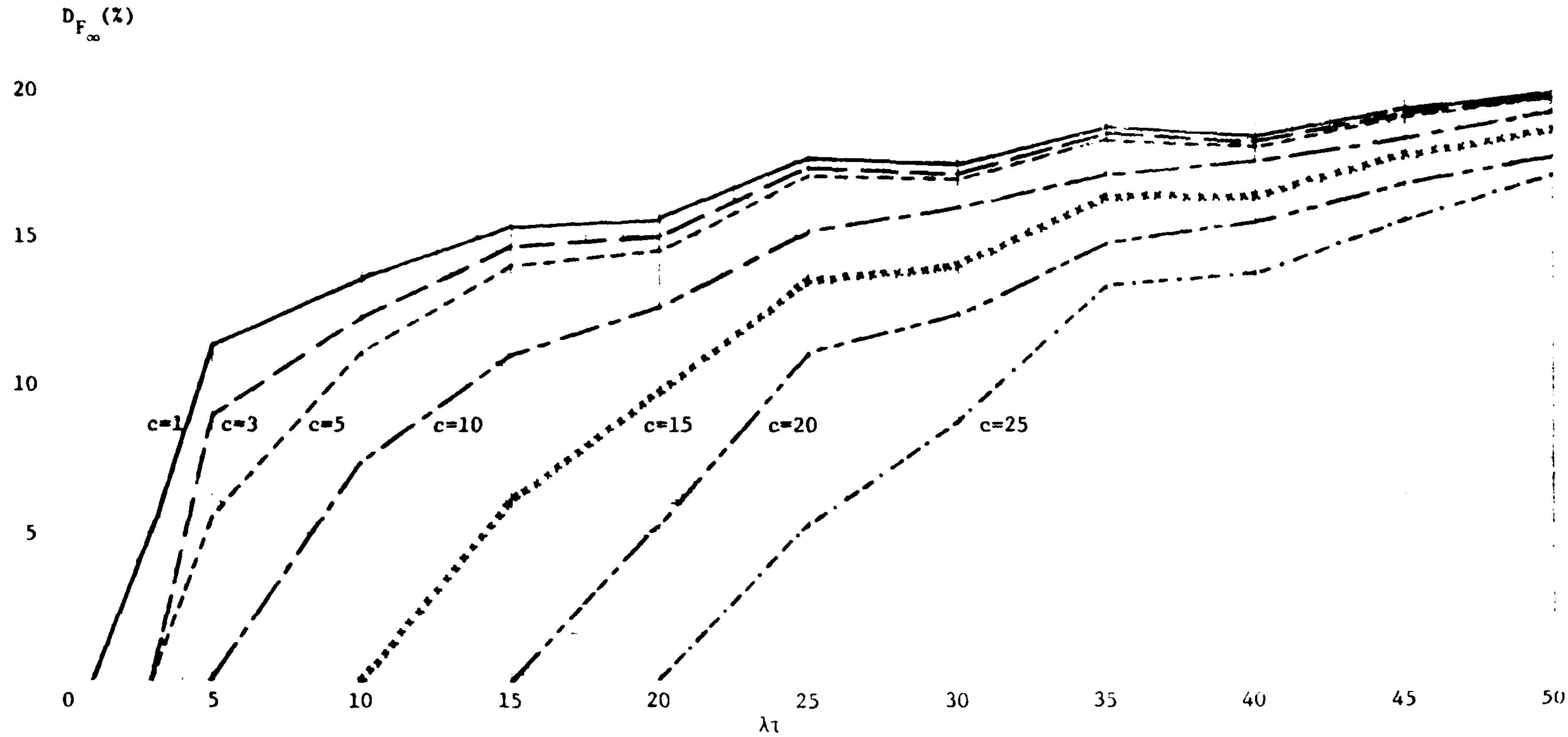


Figure 19.--D_{F_∞} vs. $\lambda\tau$ for $c=1,3,5,10,15,20,25$ and $\hat{F} = 85\%$.

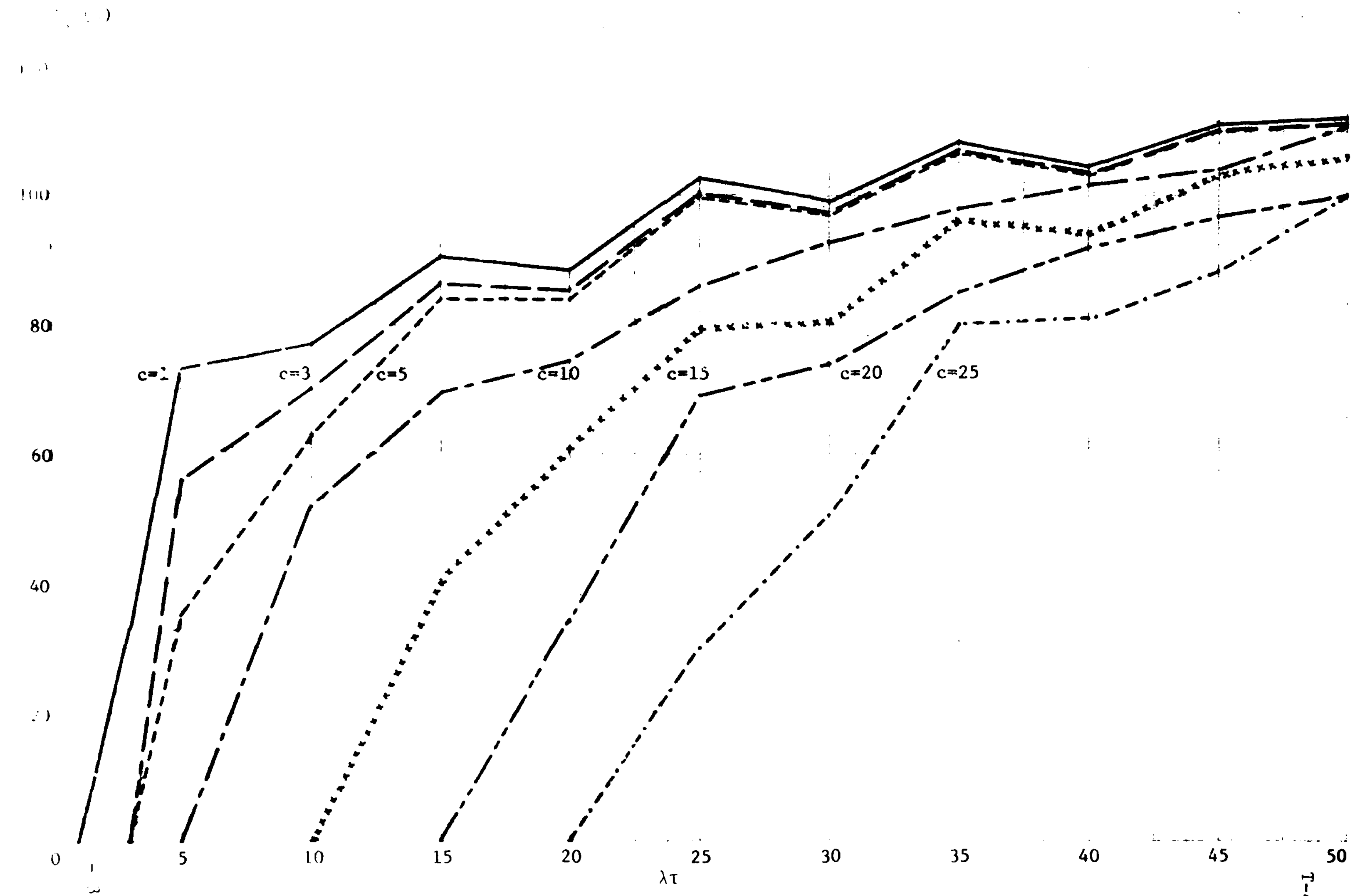


Figure 2 .--$D_{\bar{B}_\infty}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 85% .

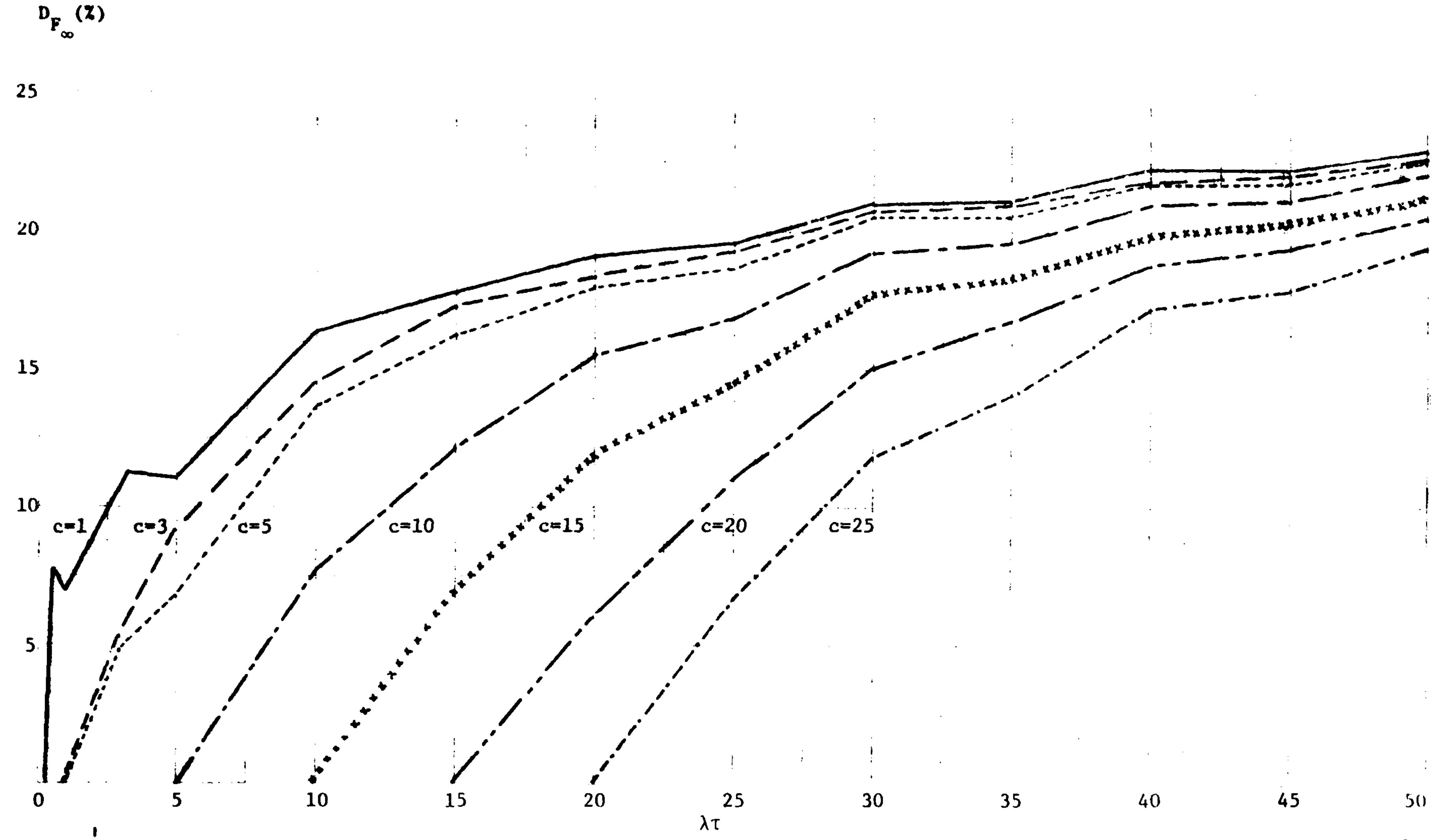


Figure 21.--D_{F_∞} vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 90% .

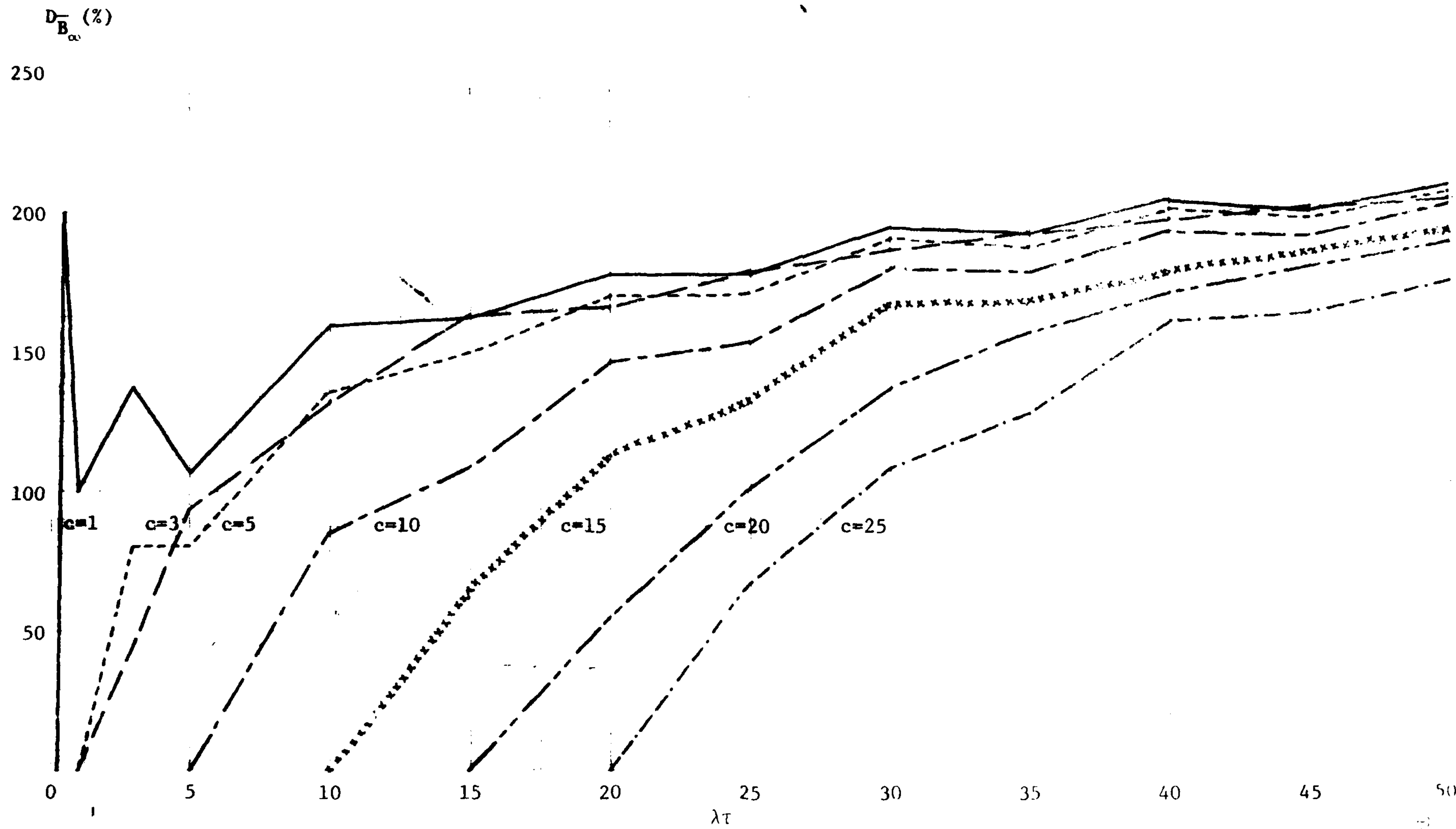


Figure 22.--$D_{\bar{B}_\infty}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 90% .

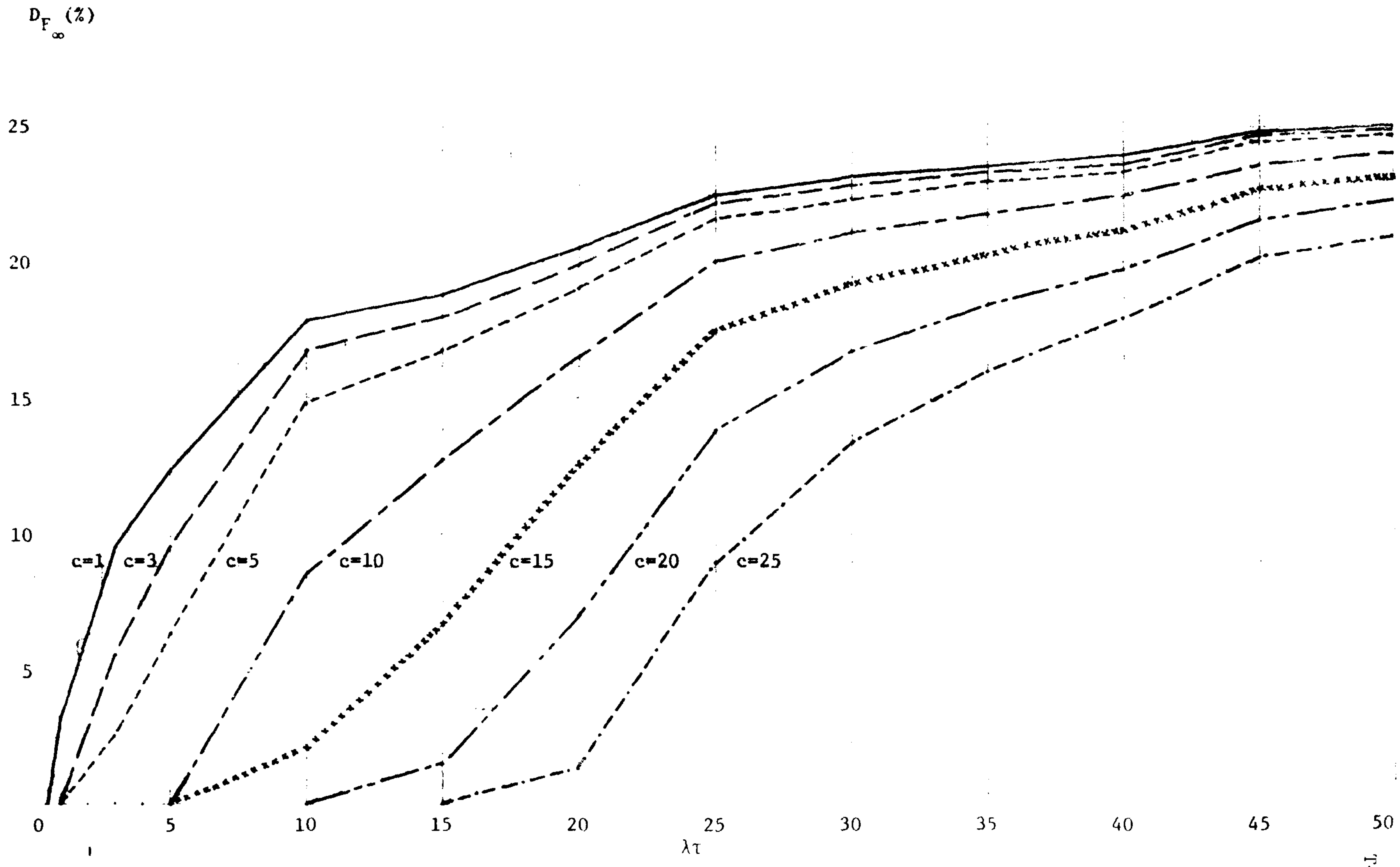


Figure 23.--D_{F_∞} vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 95% .

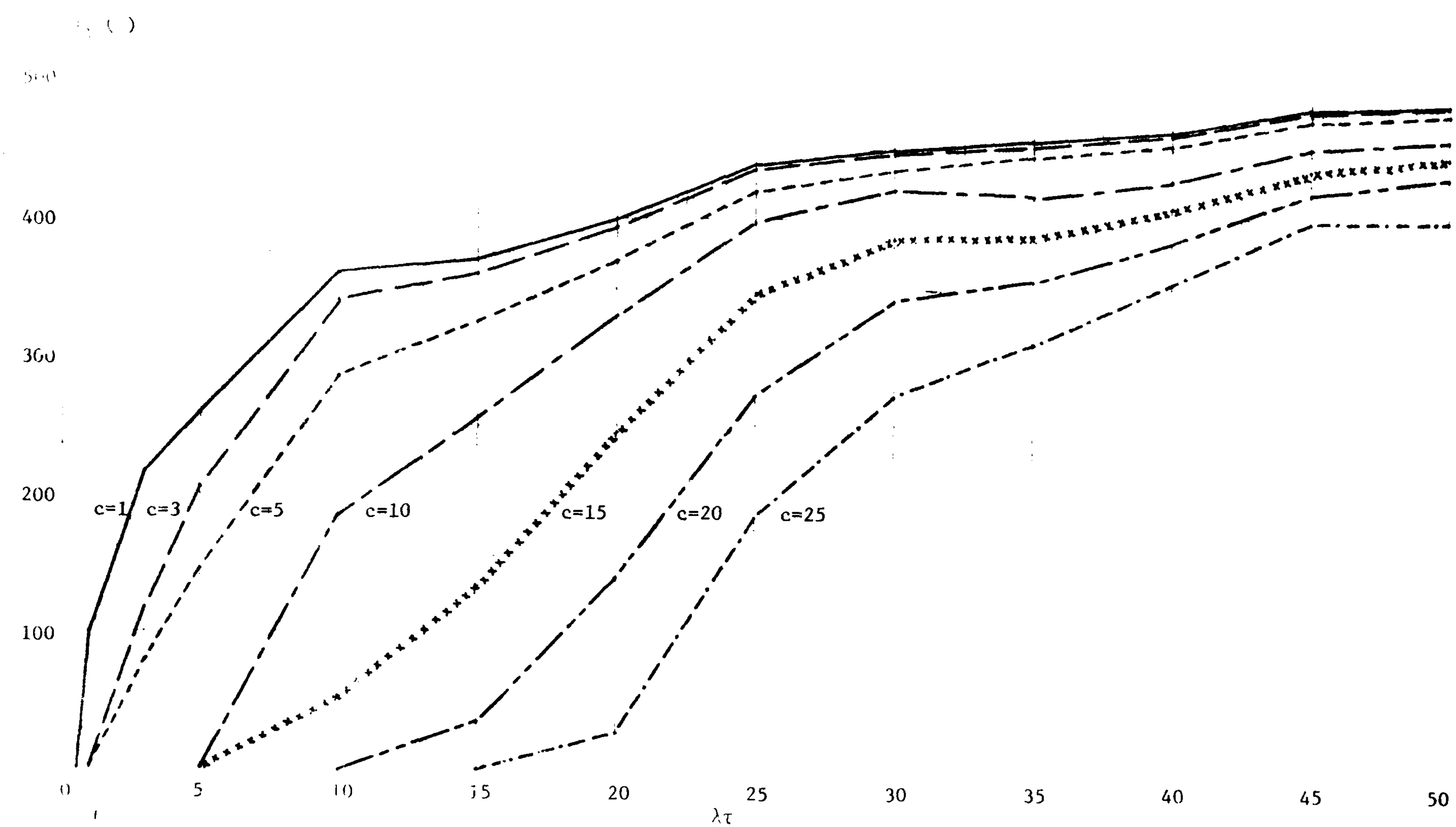


Figure 24.--$D_{\bar{B}_\alpha}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 95% .

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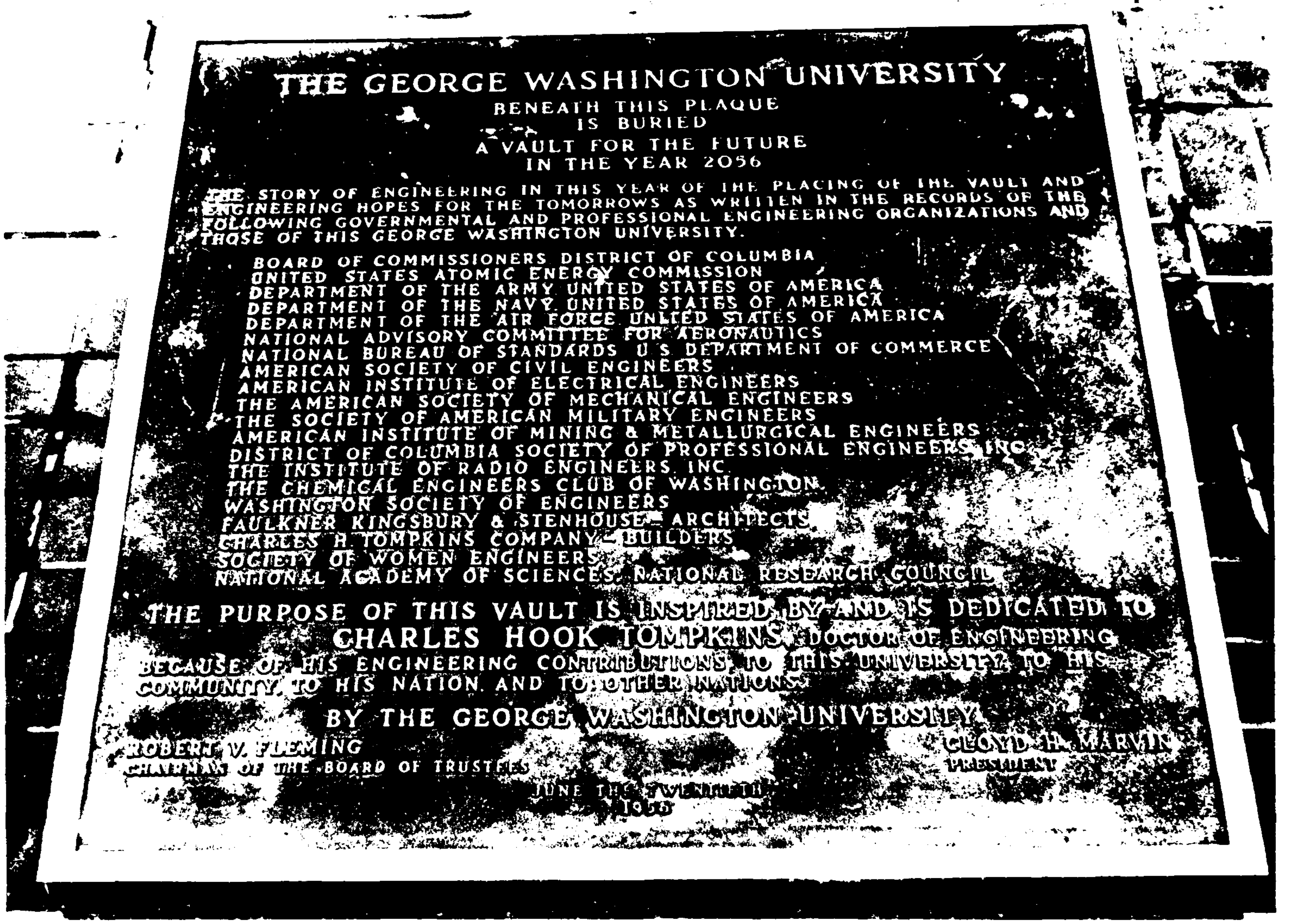

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